Thinking Recursively

Part IV

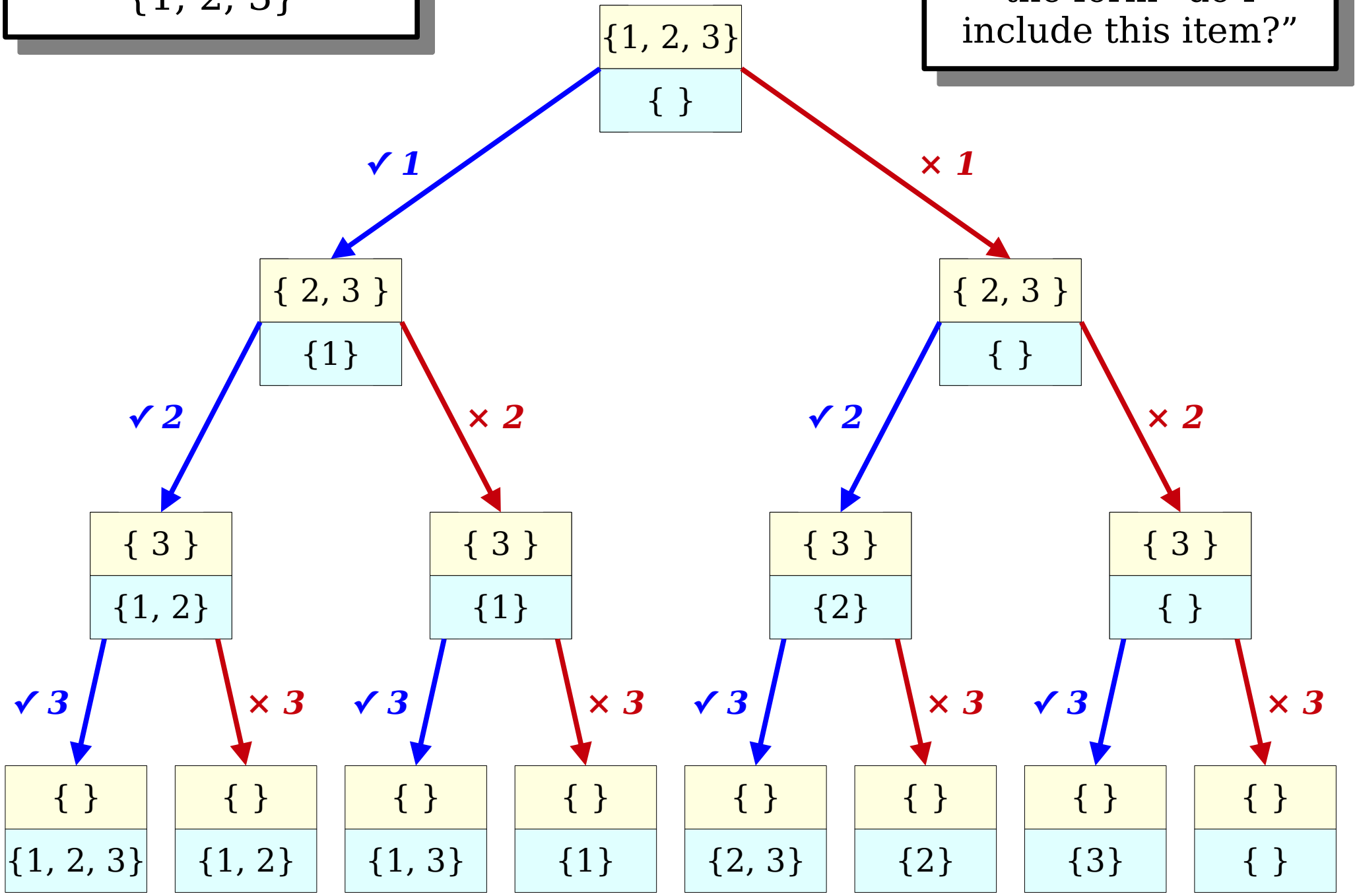
Outline for Today

- ***Recap From Last Time***
  - Where are we, again?
- ***Enumerating Combinations***
  - Forming a majority opinion.
- ***Shrinkable Words***
  - A little word puzzle!

Recap from Last Time

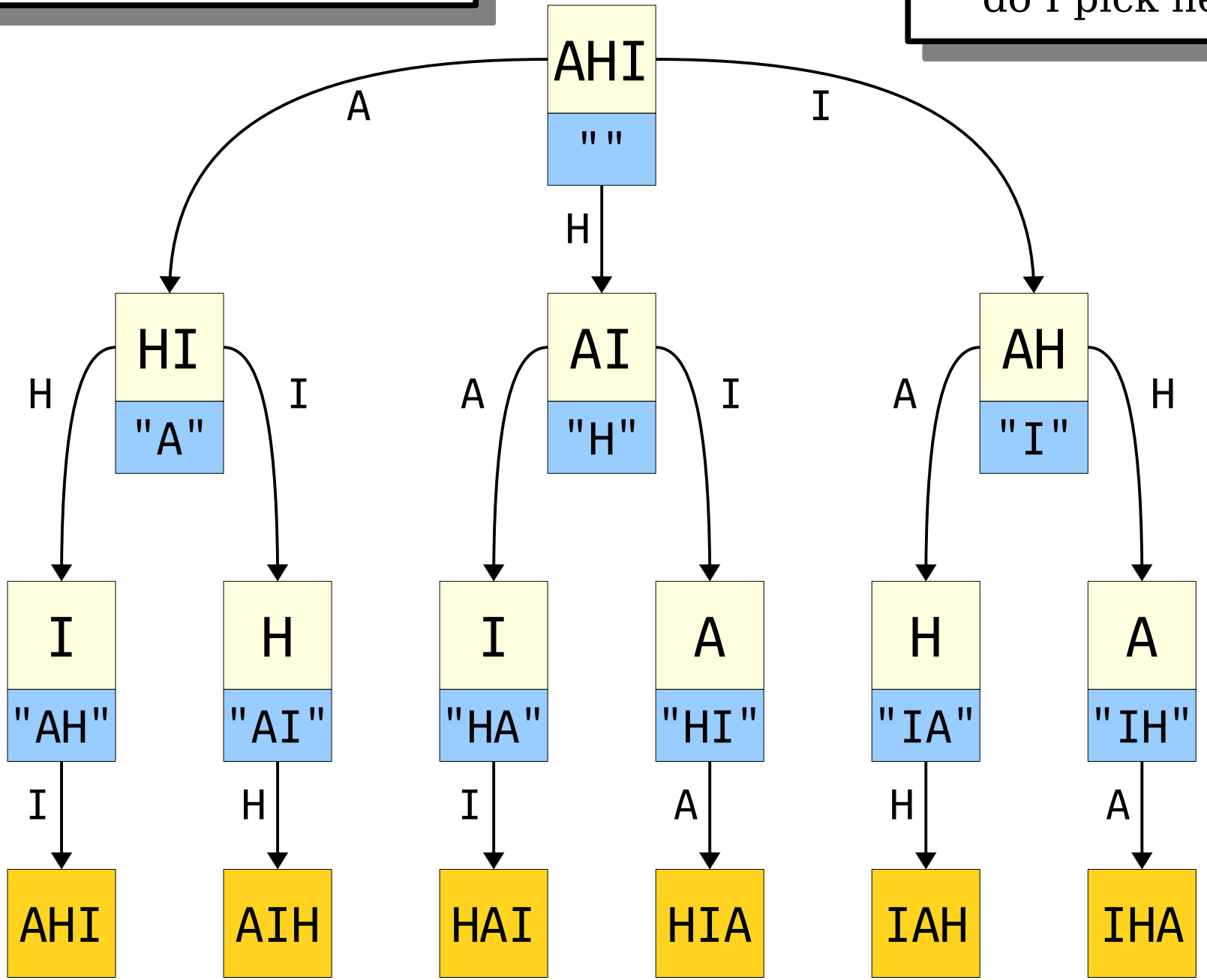
List all ***subsets*** of {1, 2, 3}

Each decision is of the form “do I include this item?”



List all ***permutations*** of {A, H, I}

Each decision is of the form “which item do I pick next?”



New Stuff!

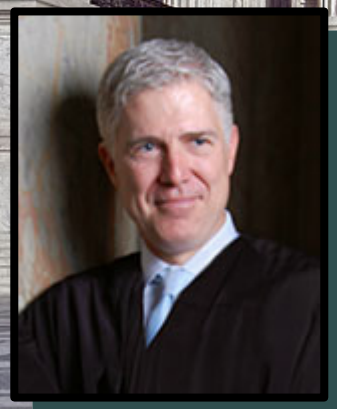
Enumerating Combinations

You need at least five US Supreme Court justices to agree to set a precedent.

What are all the ways you can pick five justices from the US Supreme Court?






Generating Combinations

- Suppose that we want to find every way to choose exactly ***one*** element from a set.
- We could do something like this:

```
for (int x: mySet) {
    cout << x << endl;
}
```

Generating Combinations

- Suppose that we want to find every way to choose exactly ***two*** elements from a set.
- We could do something like this:

```
for (int x: mySet) {
  for (int y: mySet) {
    if (x < y) {
       cout << x << ", " << y << endl;
    }
  }
}
```

Generating Combinations

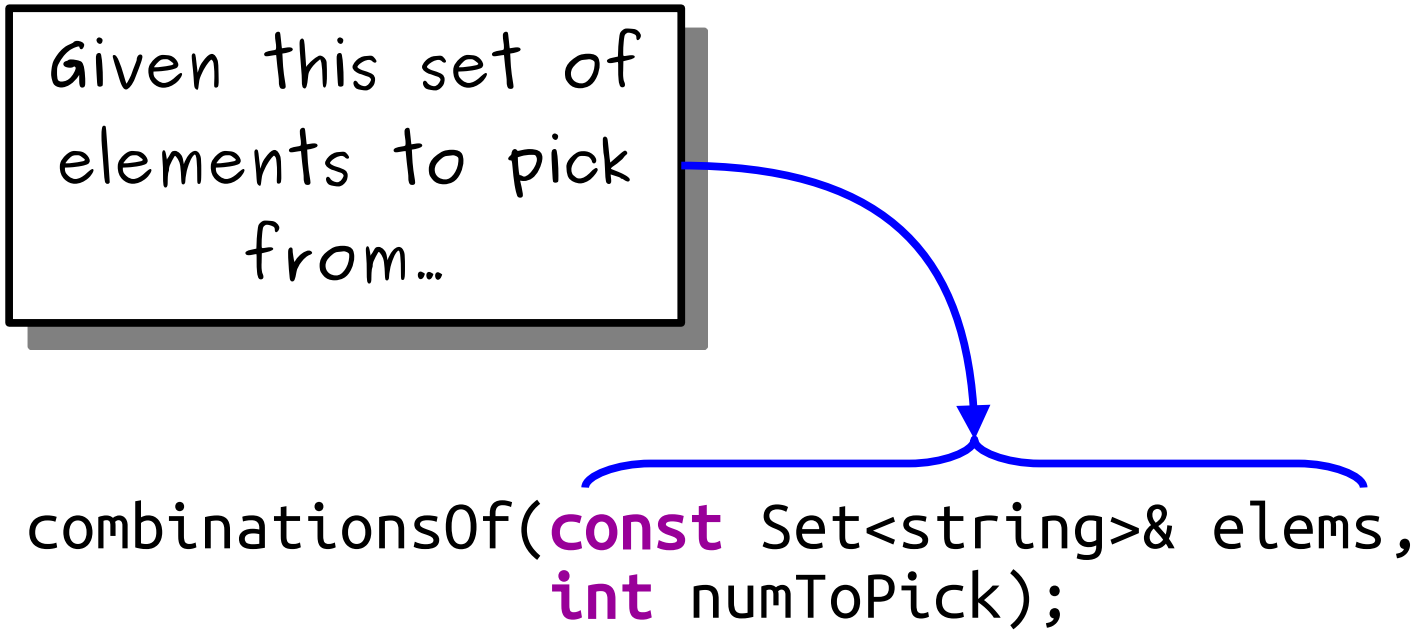
- Suppose that we want to find every way to choose exactly ***three*** elements from a set.
- We could do something like this:

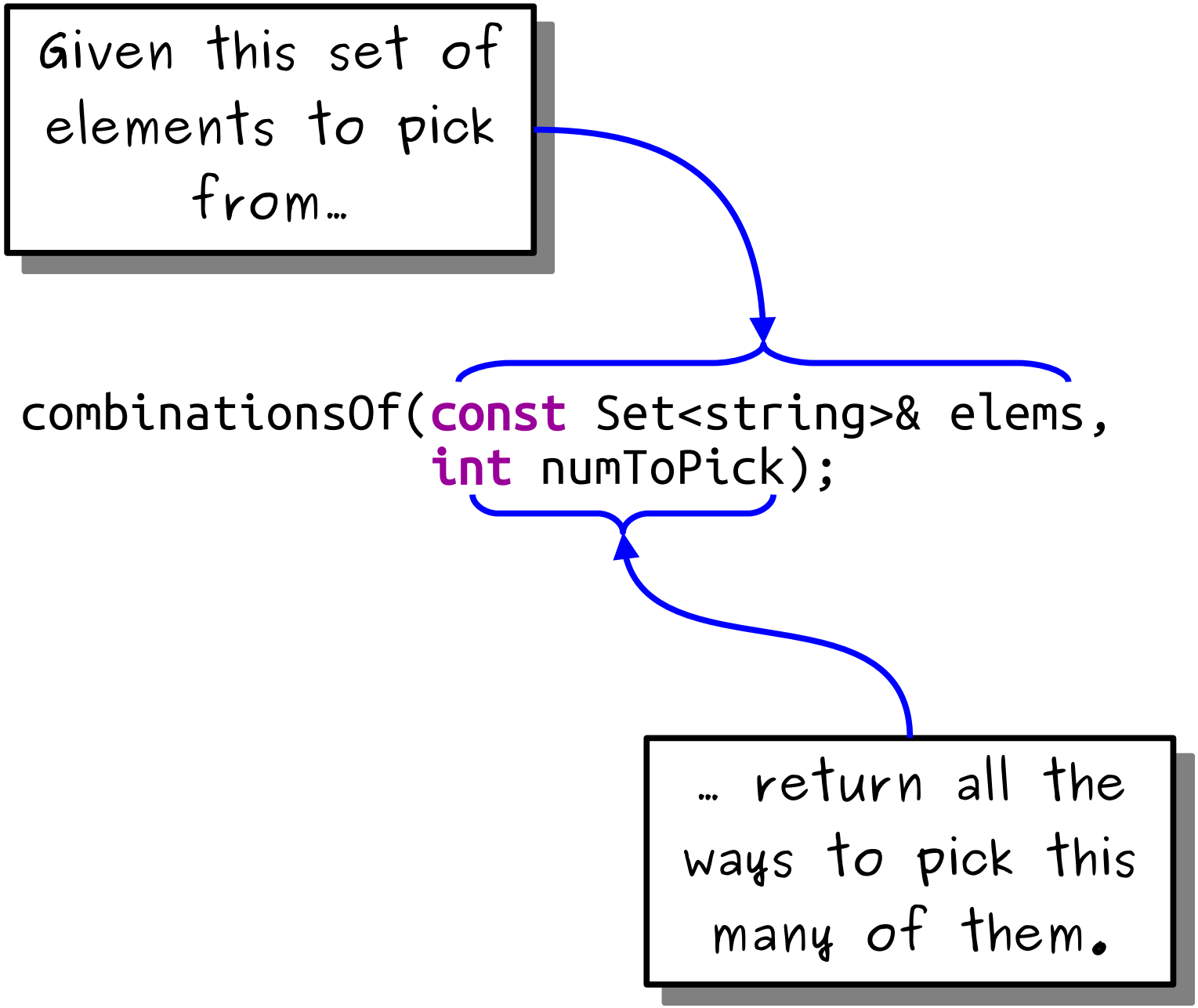
```
for (int x: mySet) {
  for (int y: mySet) {
    for (int z: mySet) {
      if (x < y && y < z) {
         cout << x << ", " << y << ", " << z << endl;
      }
    }
  }
}
```

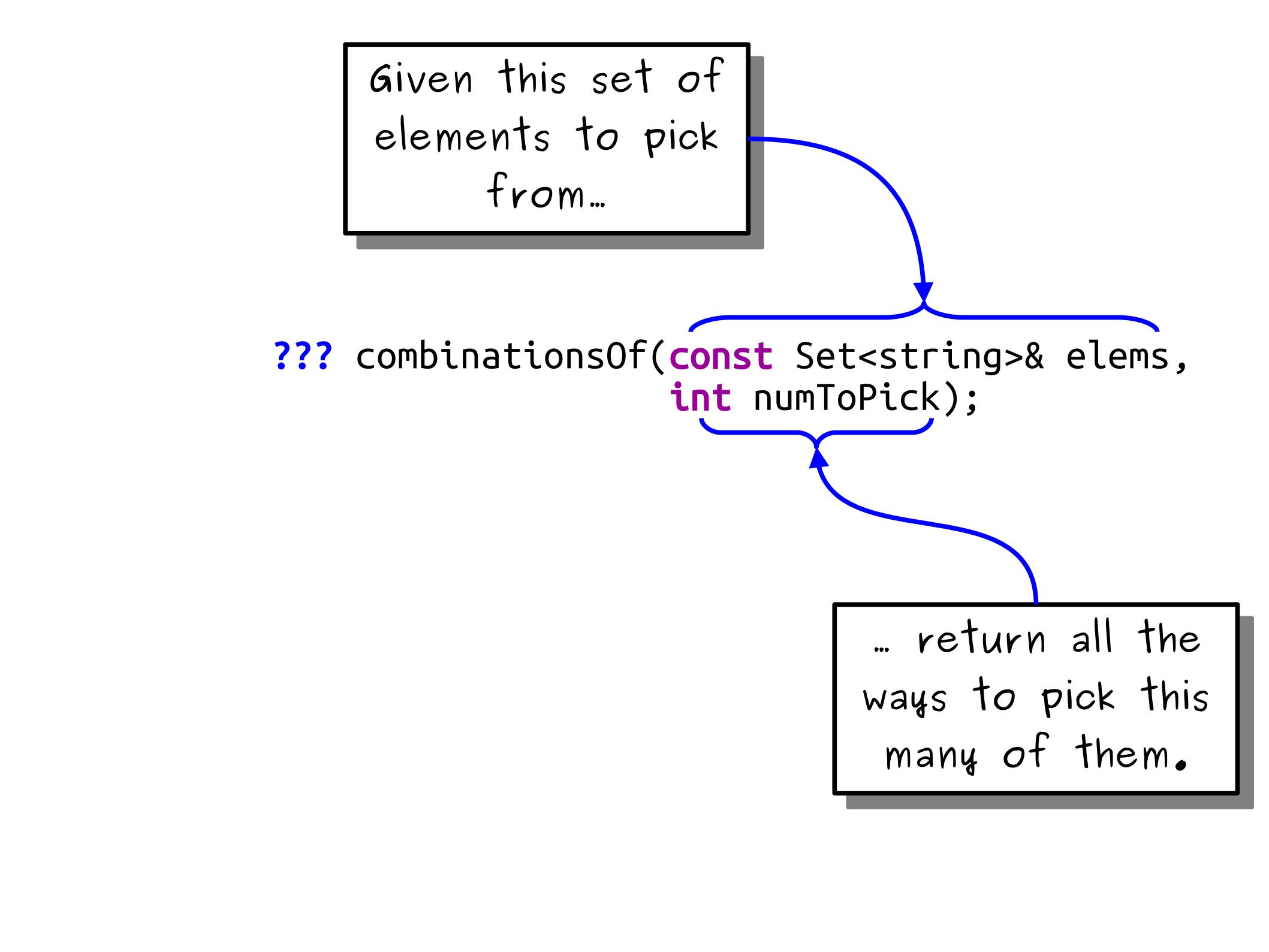
Generating Combinations

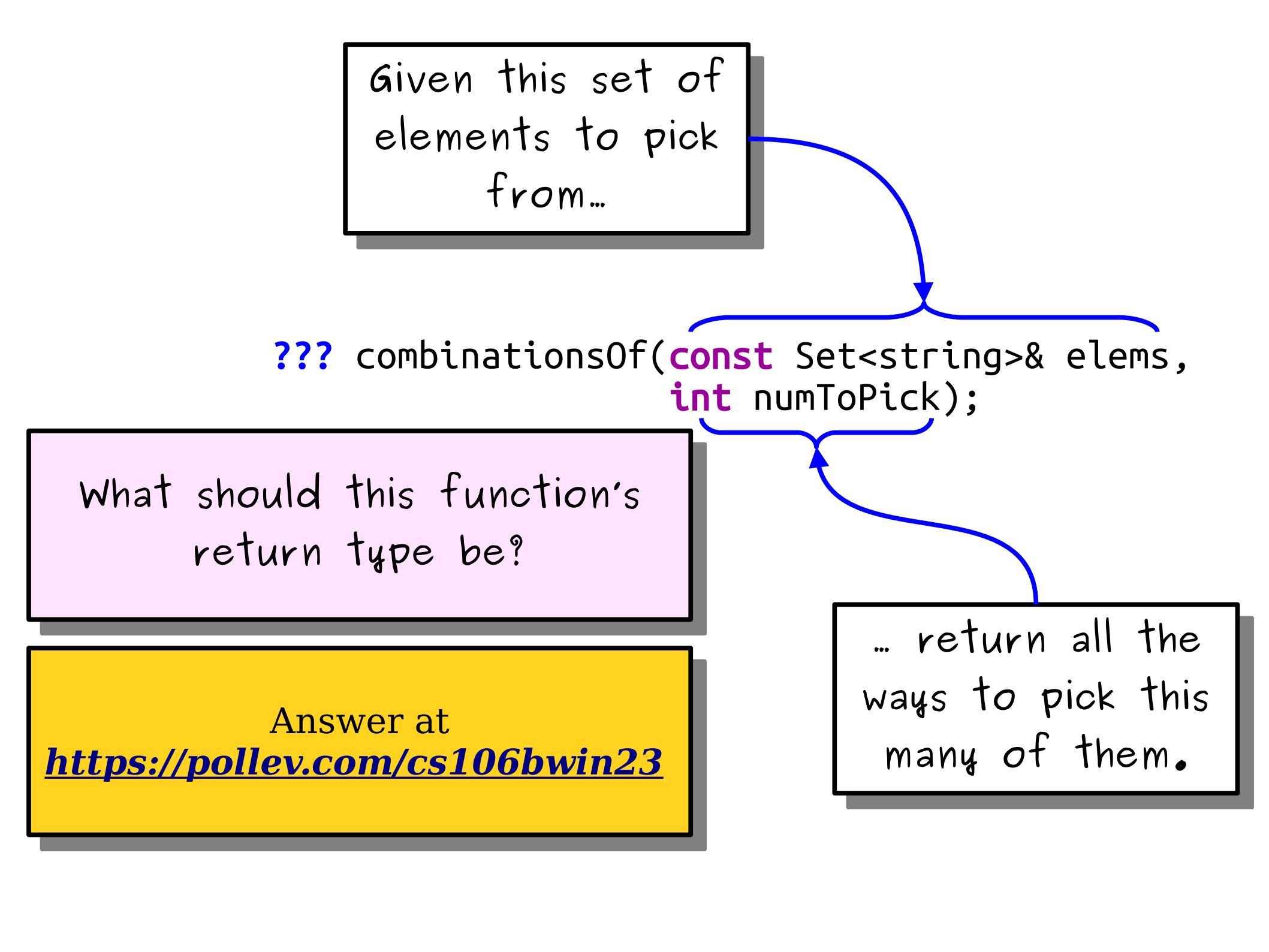
- If we know how many elements we want in advance, we can always just nest a whole bunch of loops.
- But what if we don't know in advance?
- Or we *do* know in advance, but it’s a reasonably large number and we don’t want to write a huge number of nested loops and complicated `if` statements?

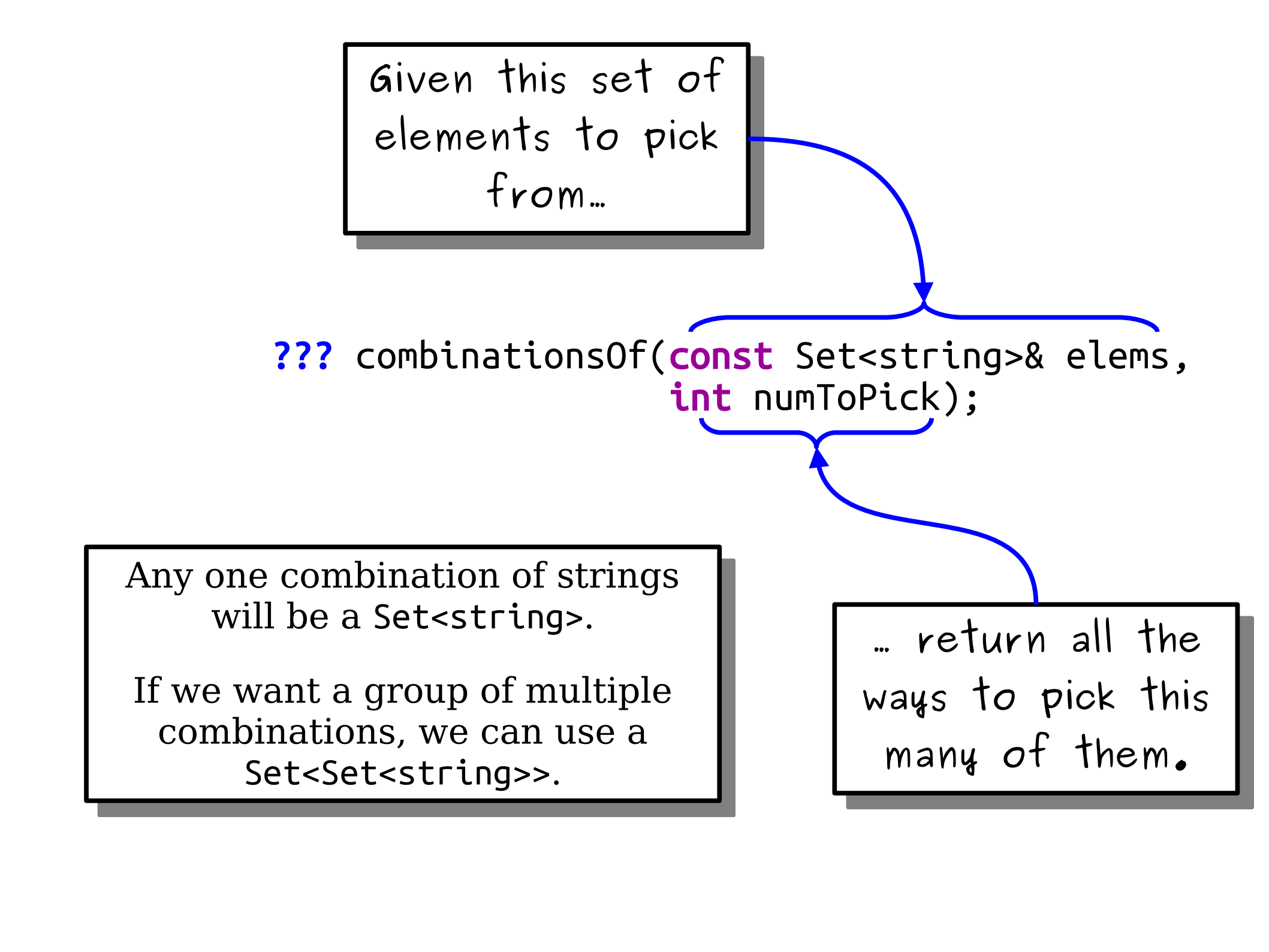
```
combinationsOf(const Set<string>& elems,
               int numToPick);
```


Given this set of elements to pick from…
combinationsOf(const Set<string>& elems,
int numToPick);


Given this set of elements to pick from…
combinationsOf(const Set<string>& elems,
int numToPick);
… return all the ways to pick this many of them.

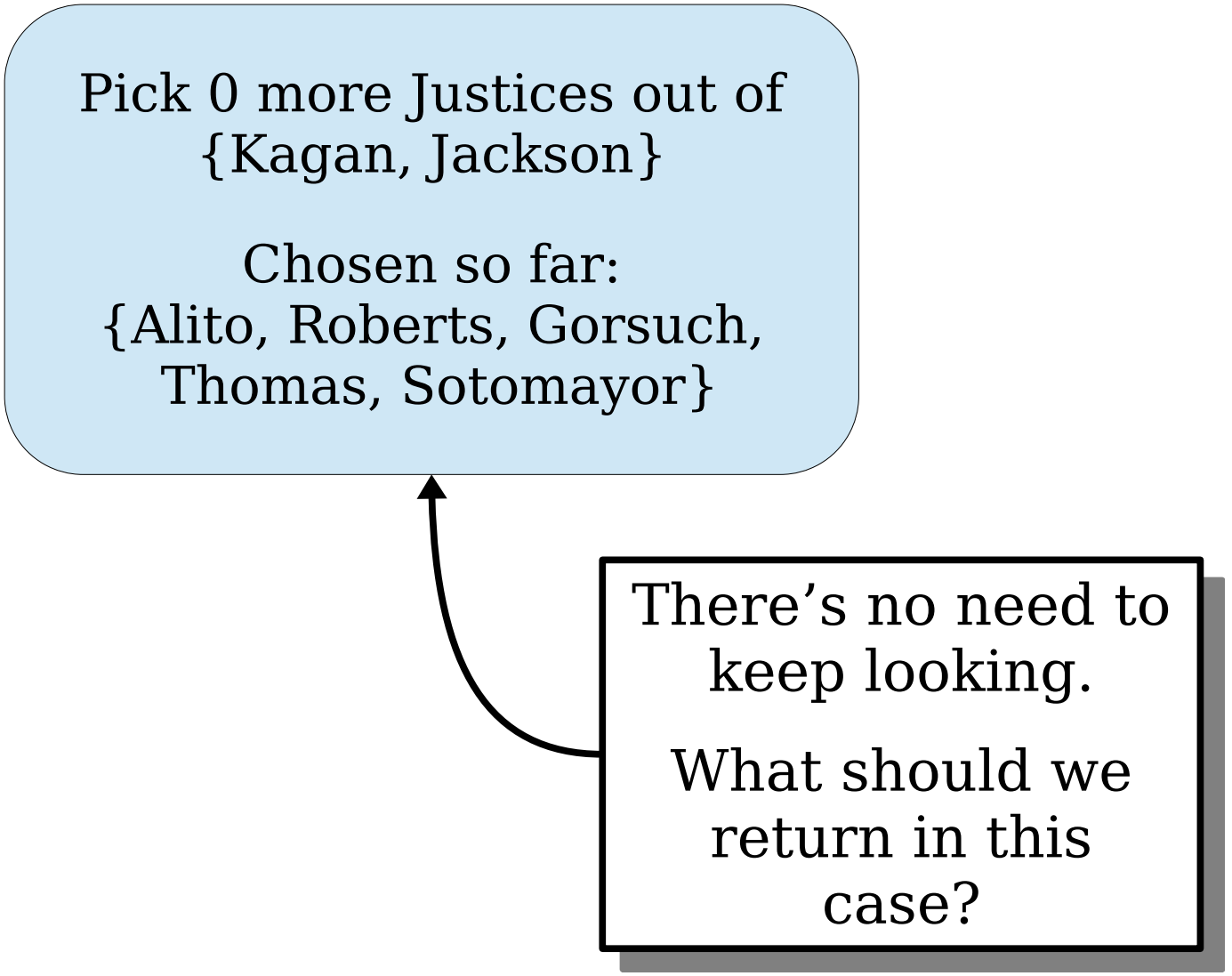

Given this set of elements to pick from…
??? combinationsOf(const Set<string>& elems,
int numToPick);
… return all the ways to pick this many of them.


Given this set of elements to pick from…
??? combinationsOf(const Set<string>& elems,
int numToPick);
What should this function's return type be?
Answer at
https://pollev.com/cs106bwin23
… return all the ways to pick this many of them.

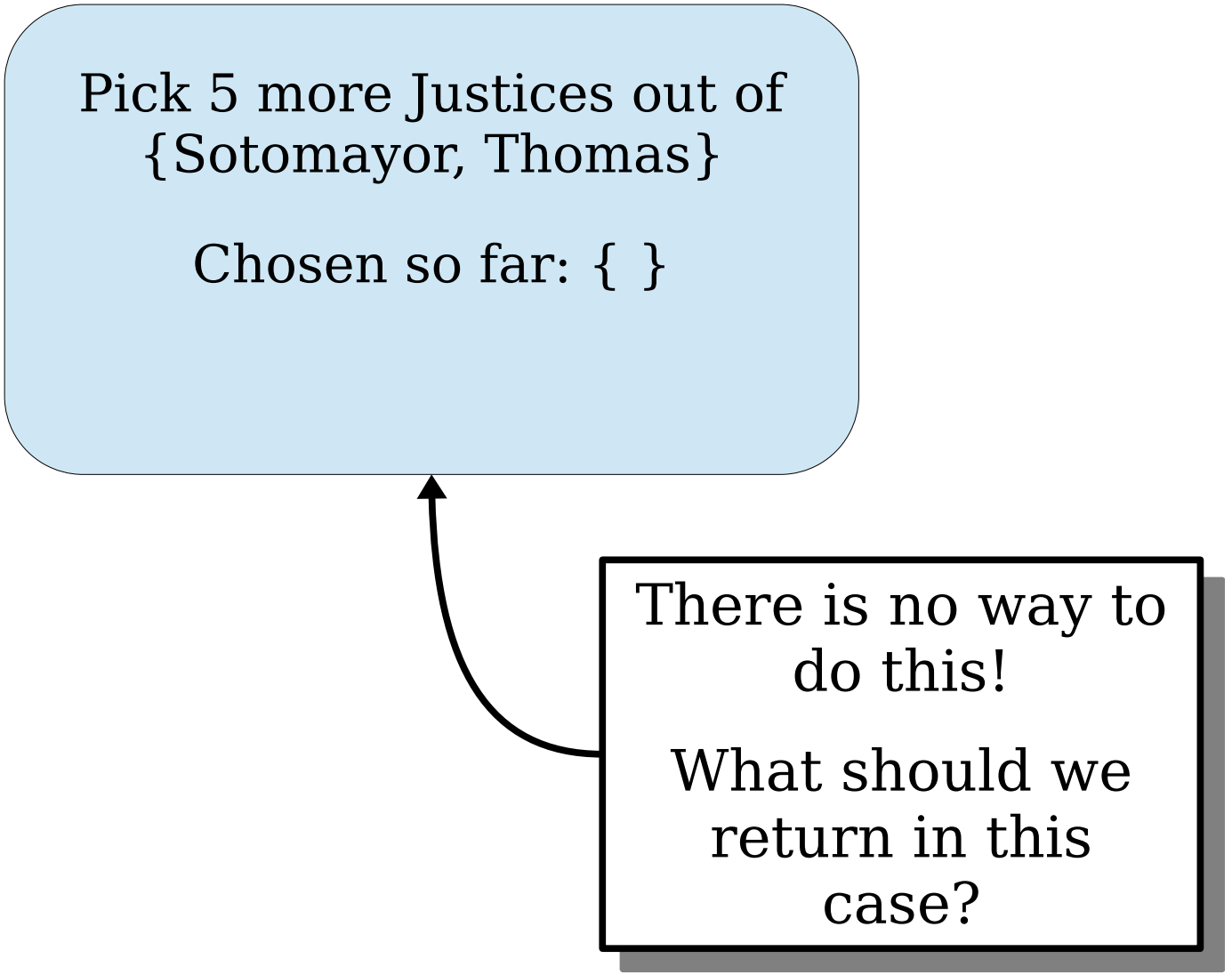

Given this set of elements to pick from…
??? combinationsOf(const Set<string>& elems,
int numToPick);
Any one combination of strings will be a Set<string>.
If we want a group of multiple combinations, we can use a Set<Set<string>>.
… return all the ways to pick this many of them.

Implementing Combinations

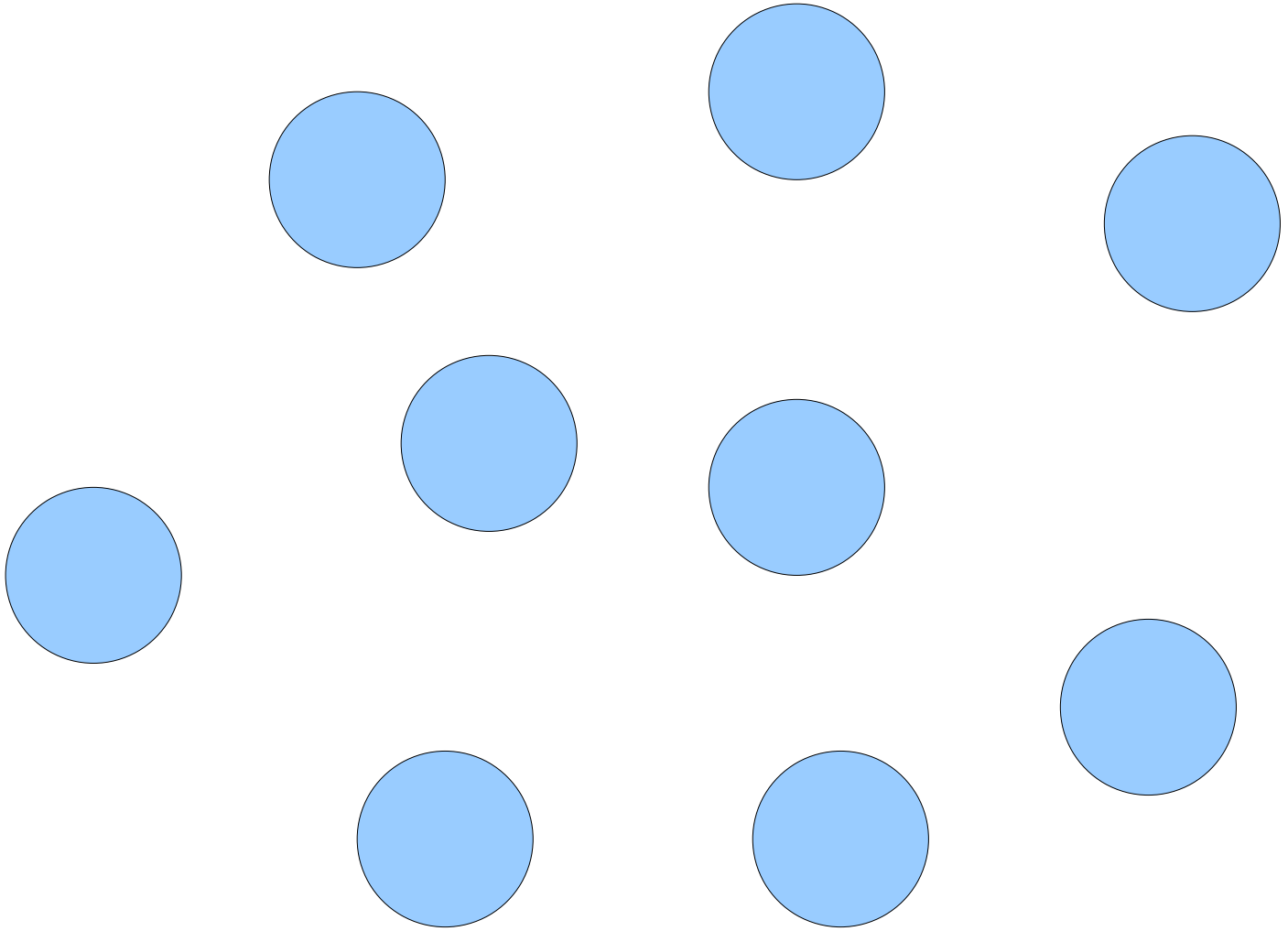
Our Base Case



Our Base Case, Part II



Generating Combinations



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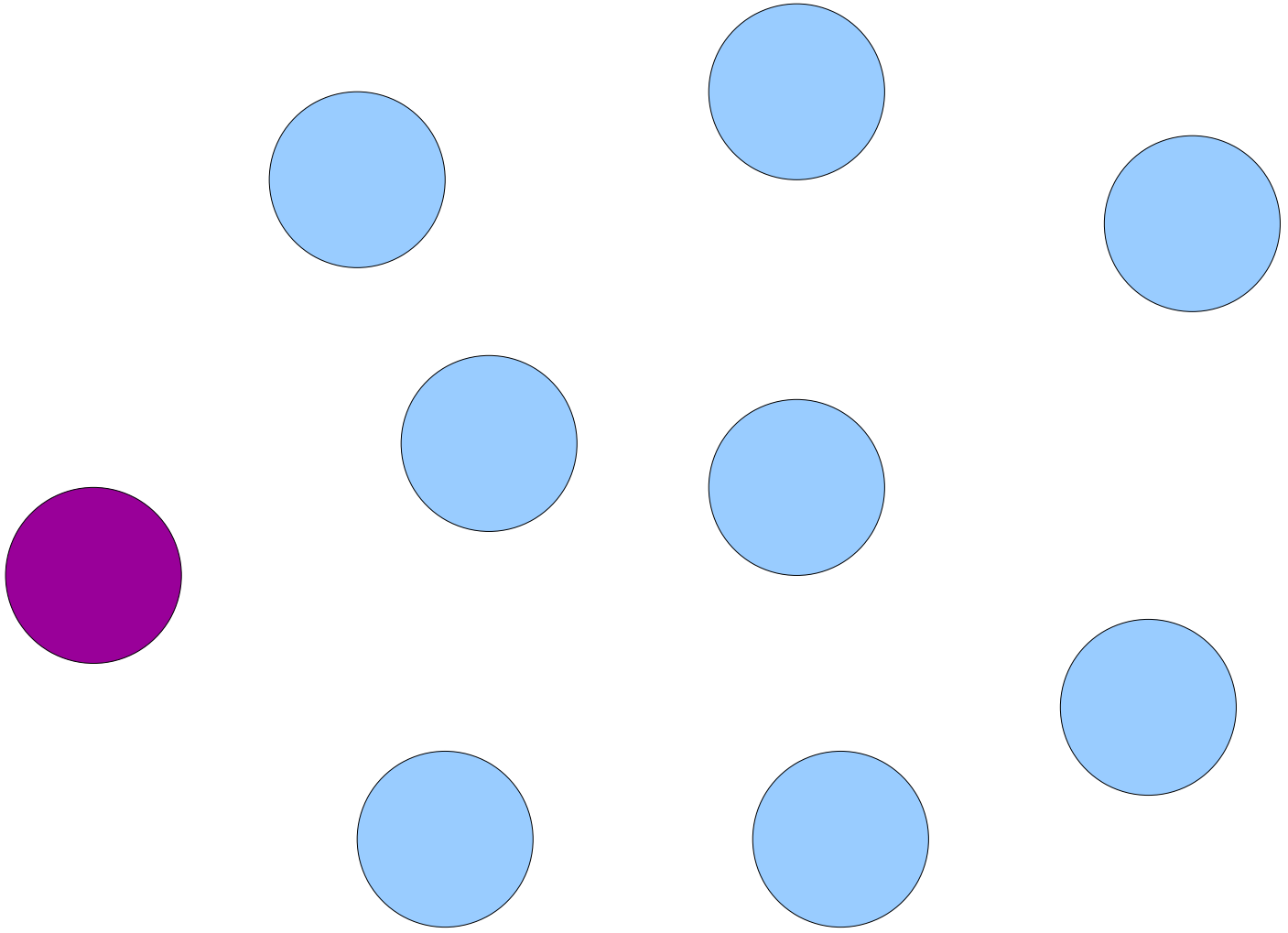

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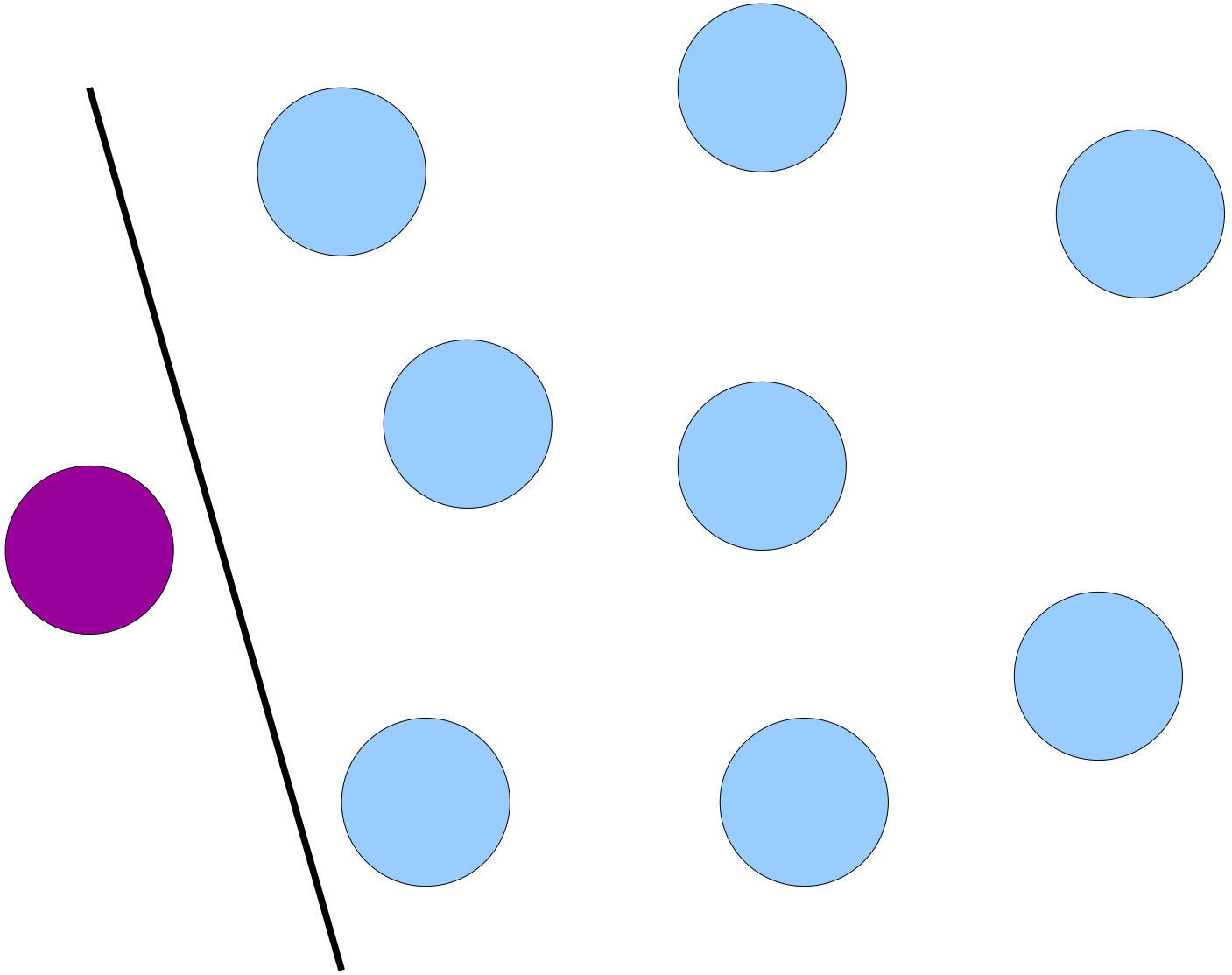
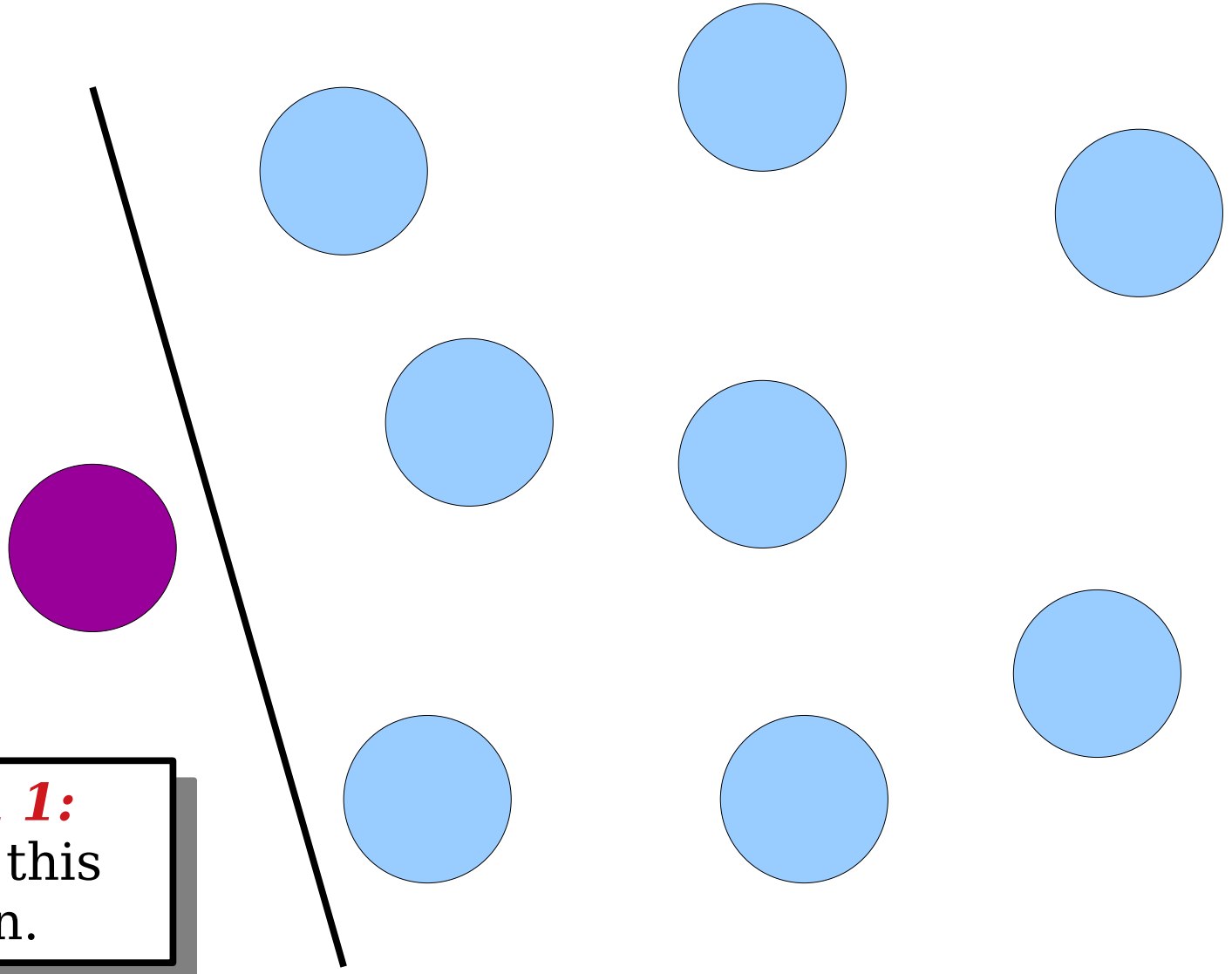
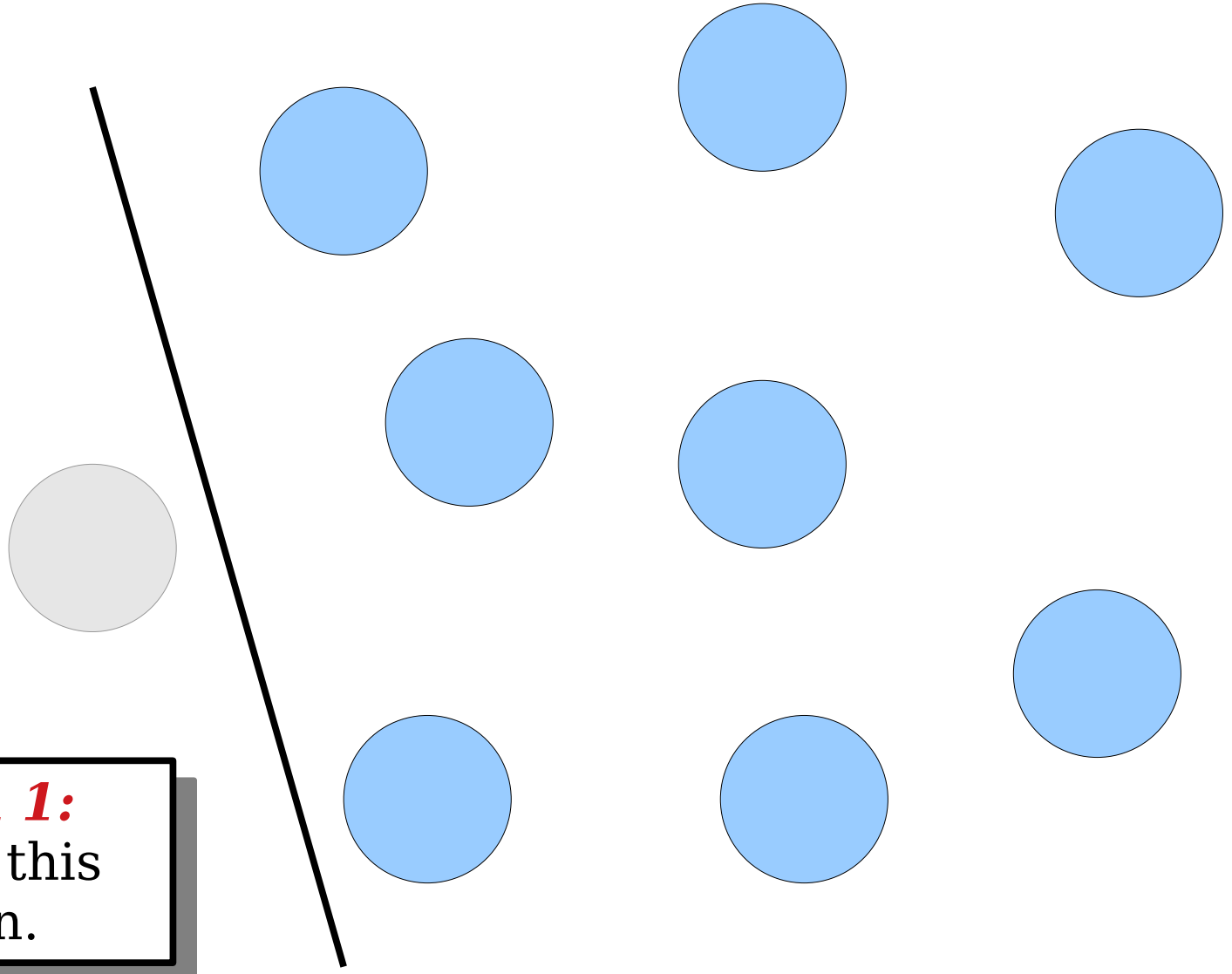


Generating Combinations



Option 1:
Exclude this person.

Generating Combinations



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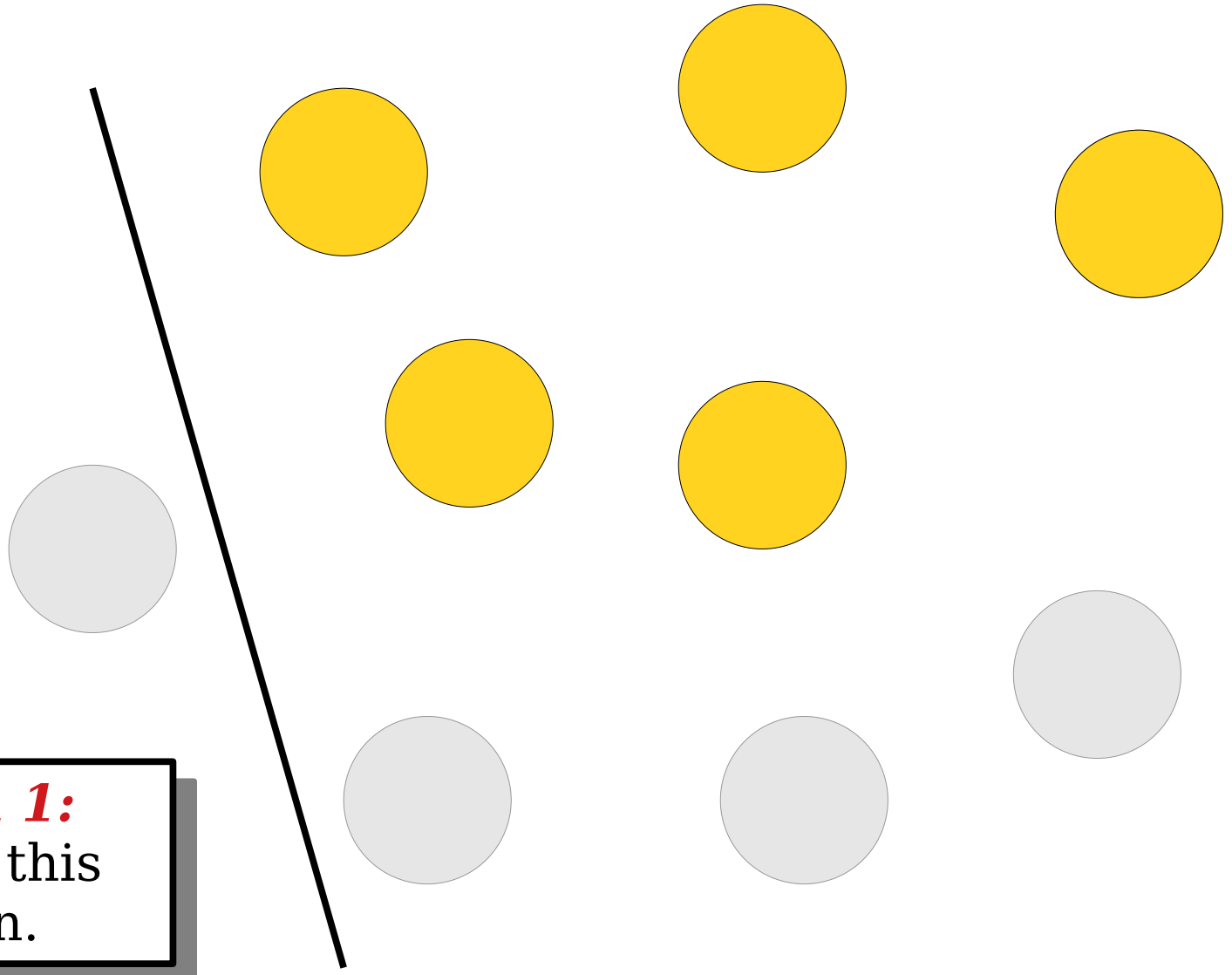

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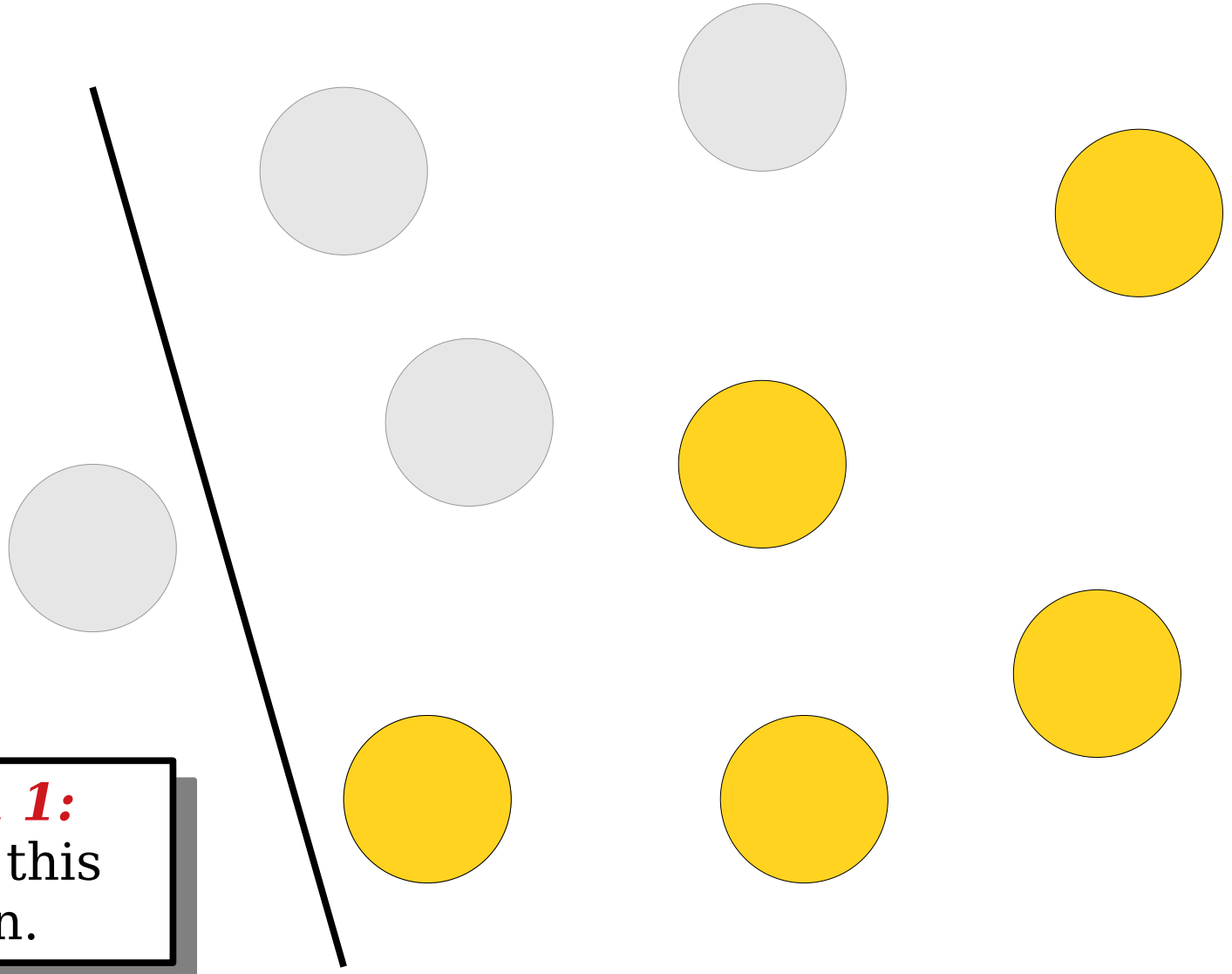

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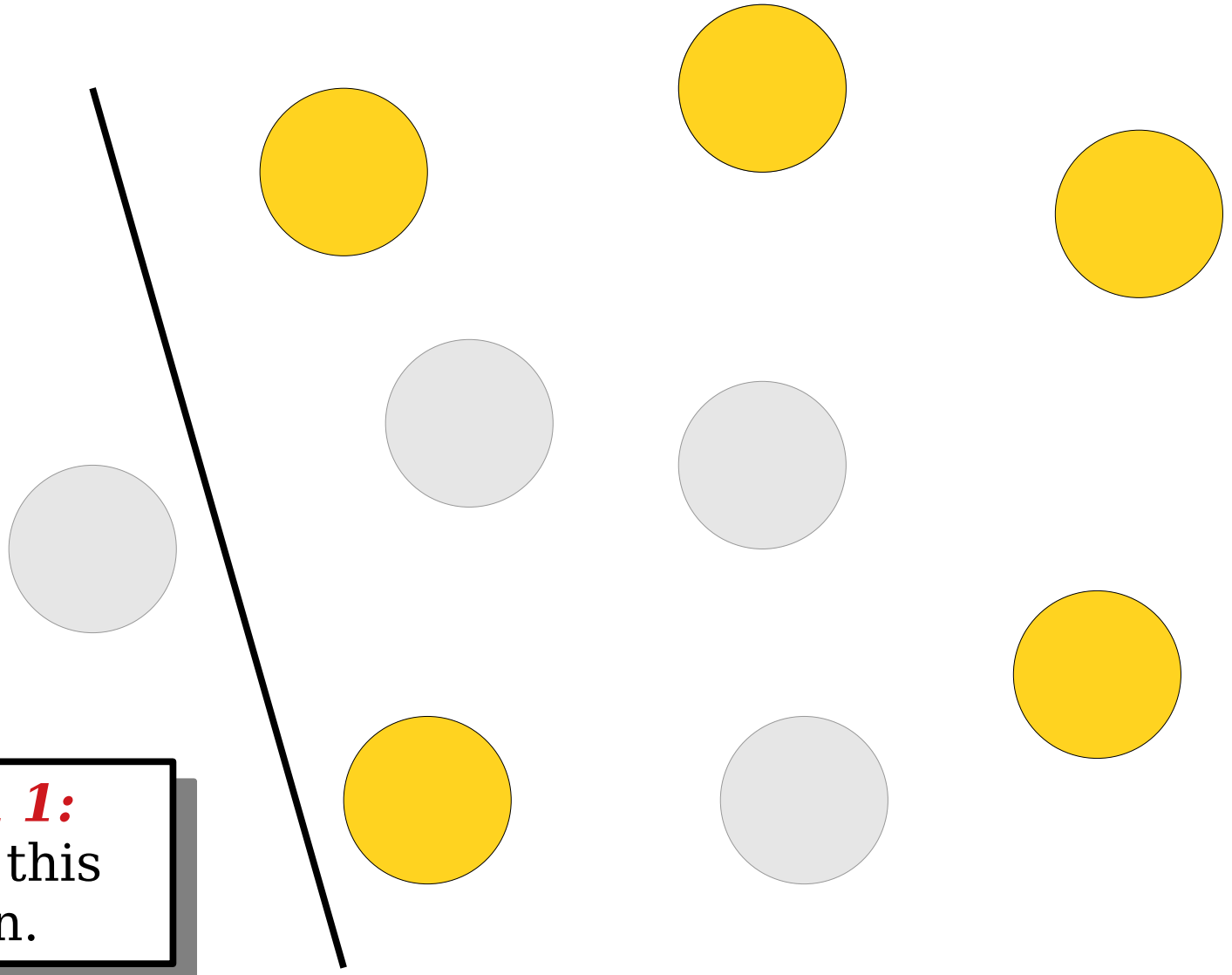

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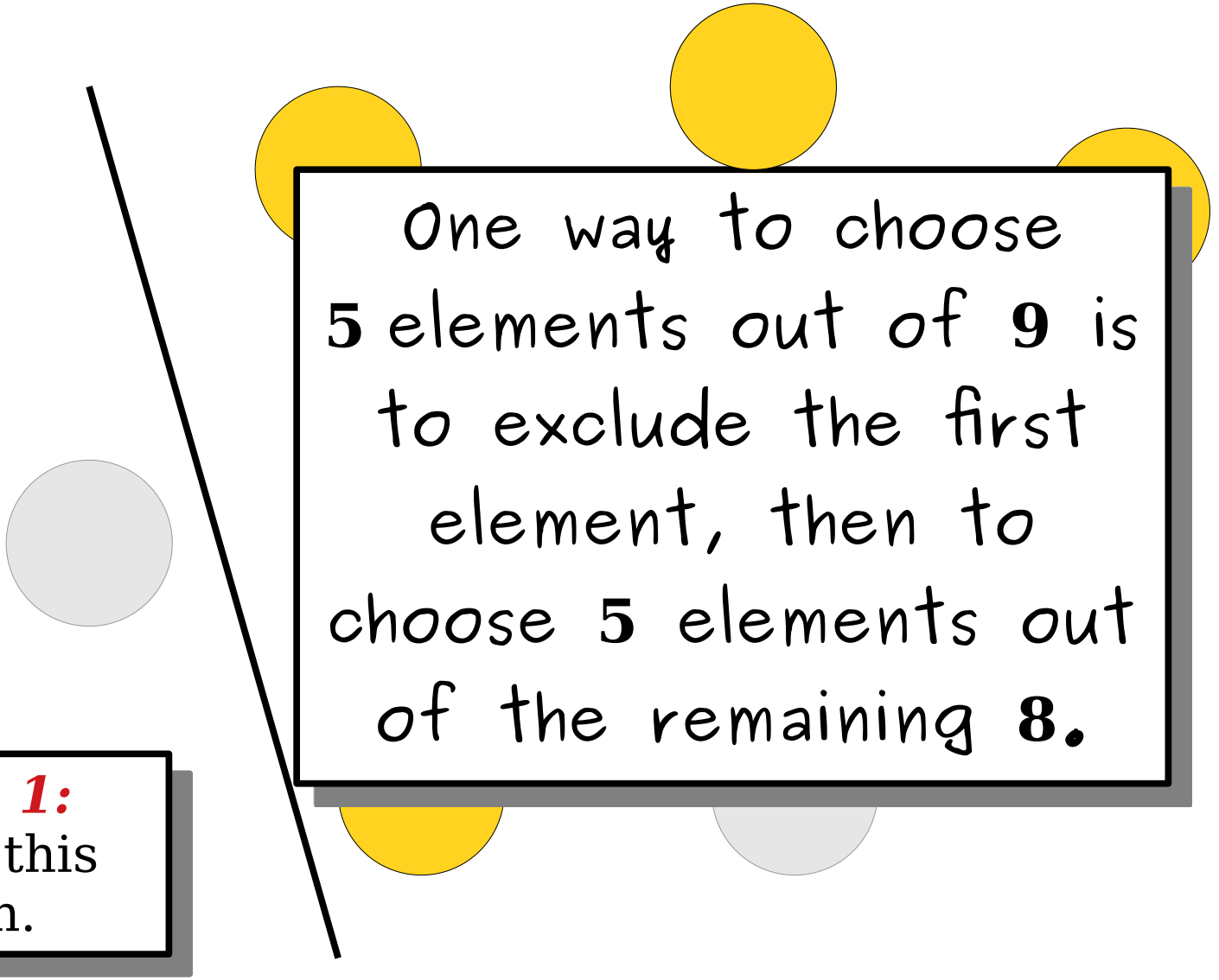

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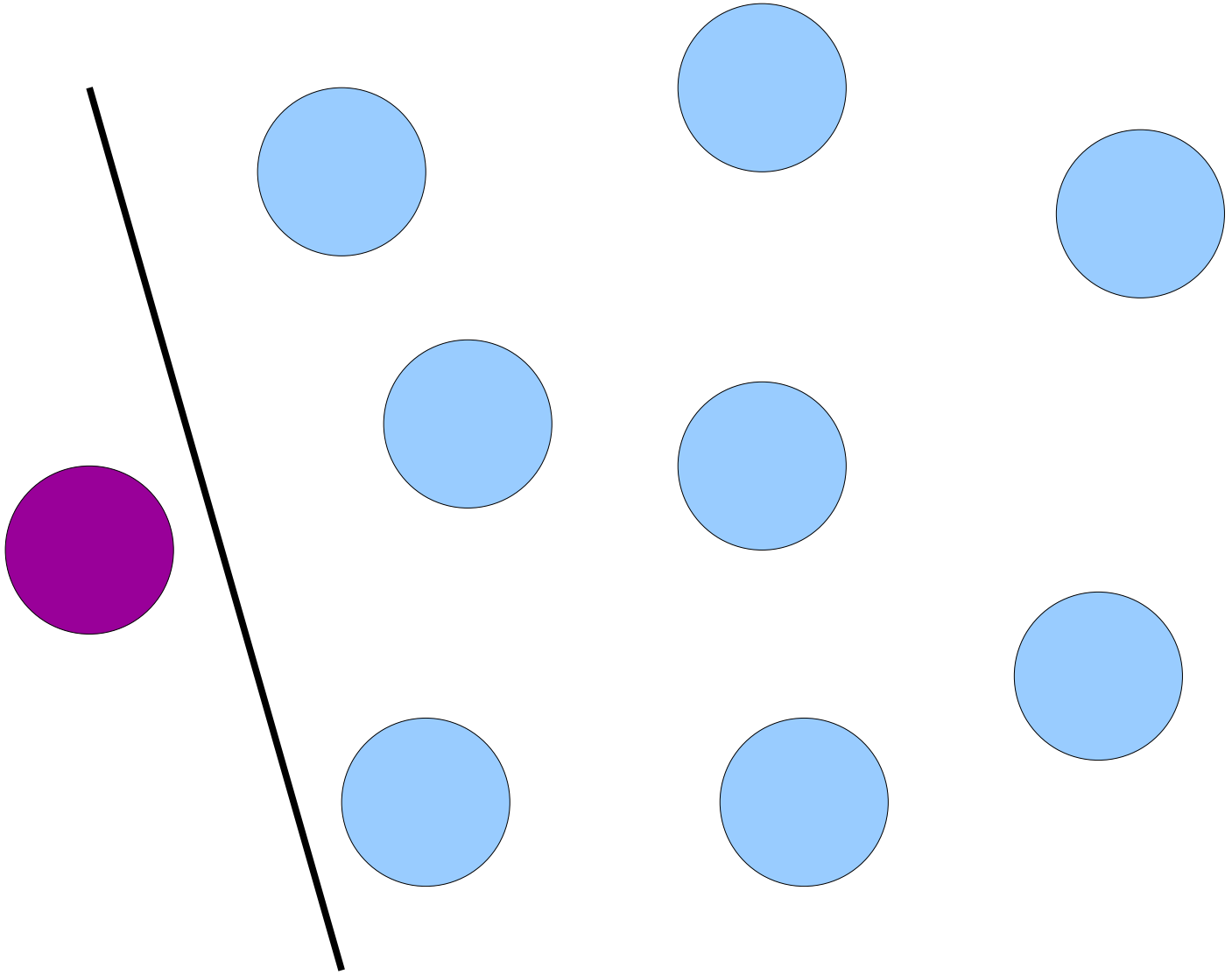
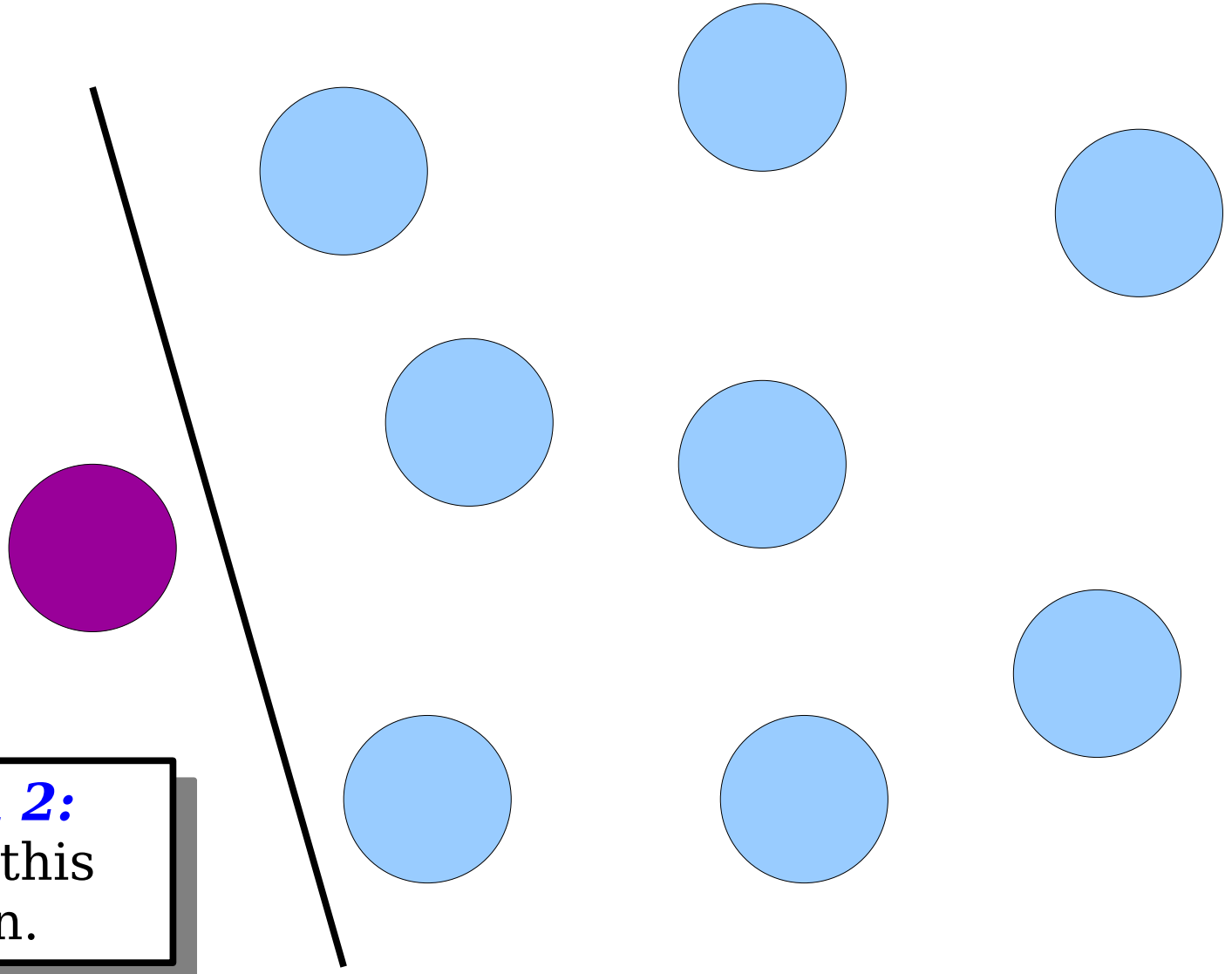
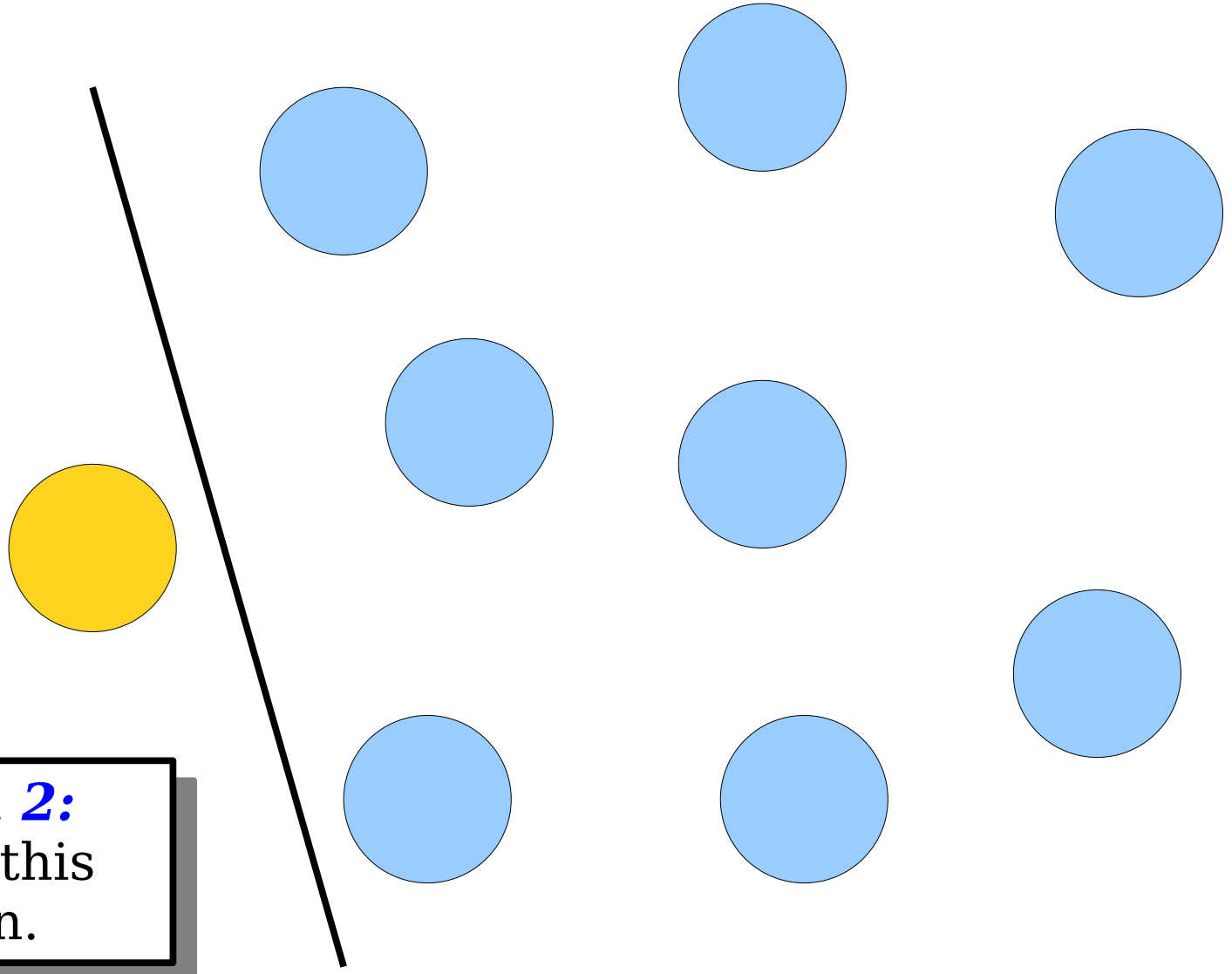


Generating Combinations



Option 2:
Include this person.

Generating Combinations



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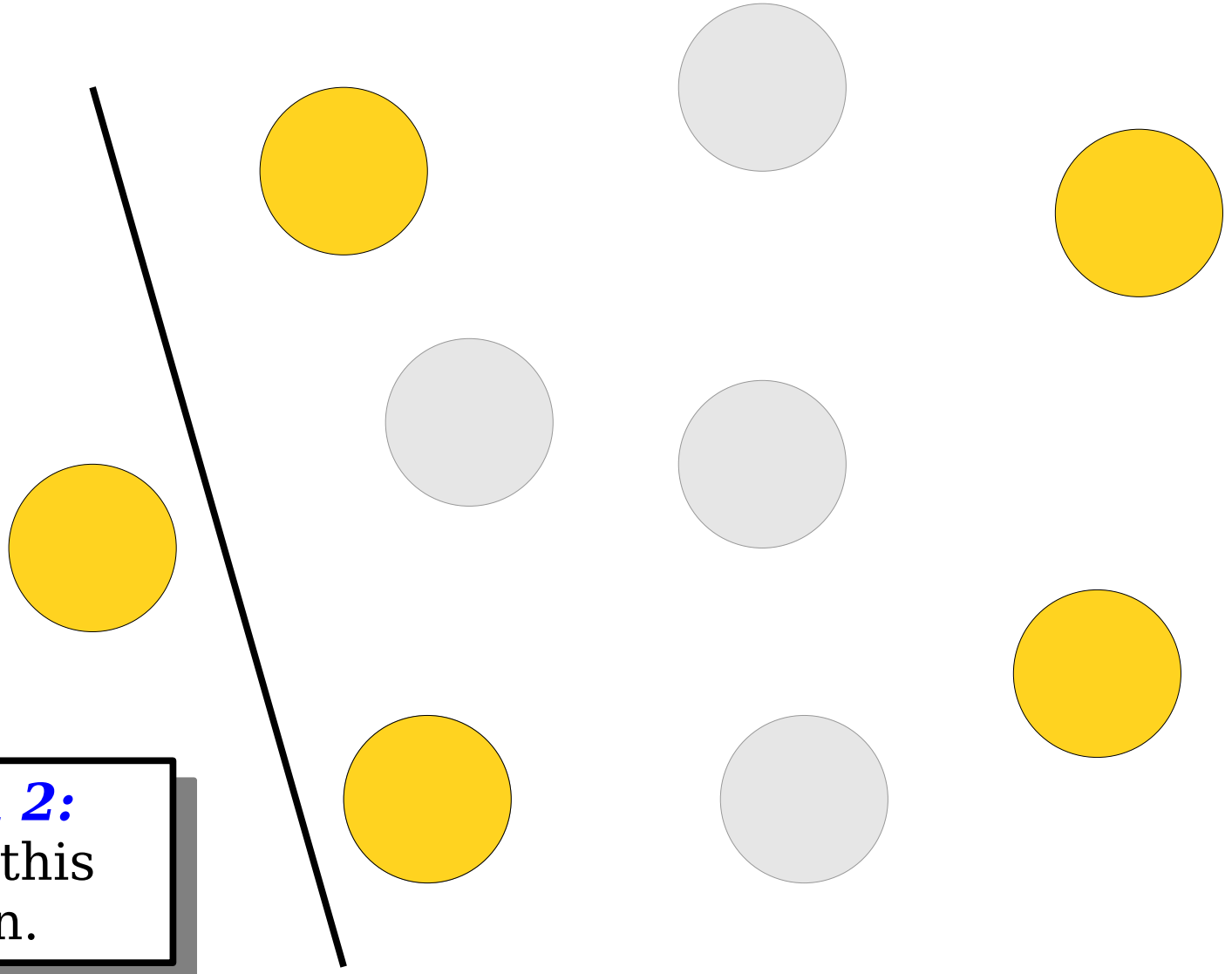

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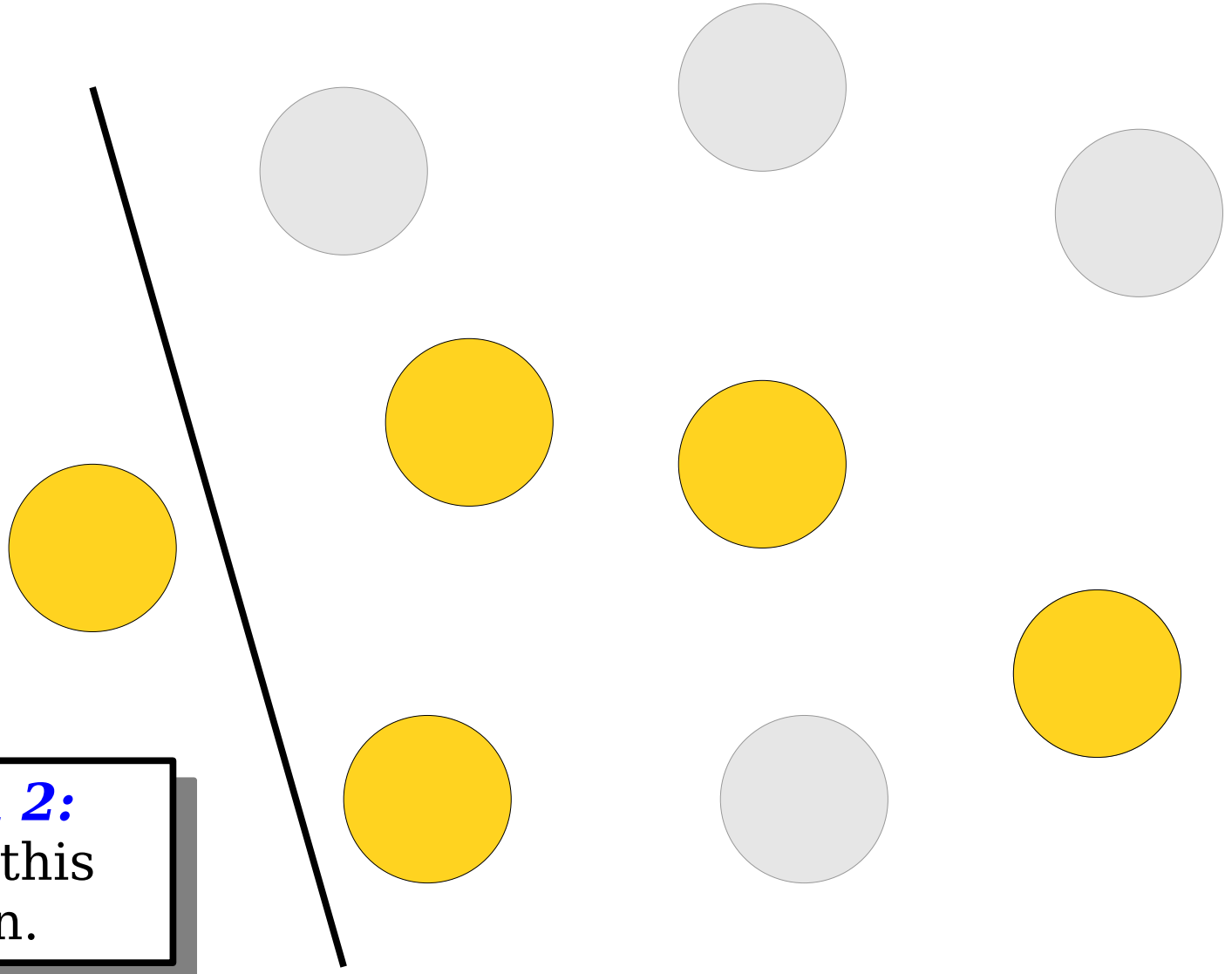

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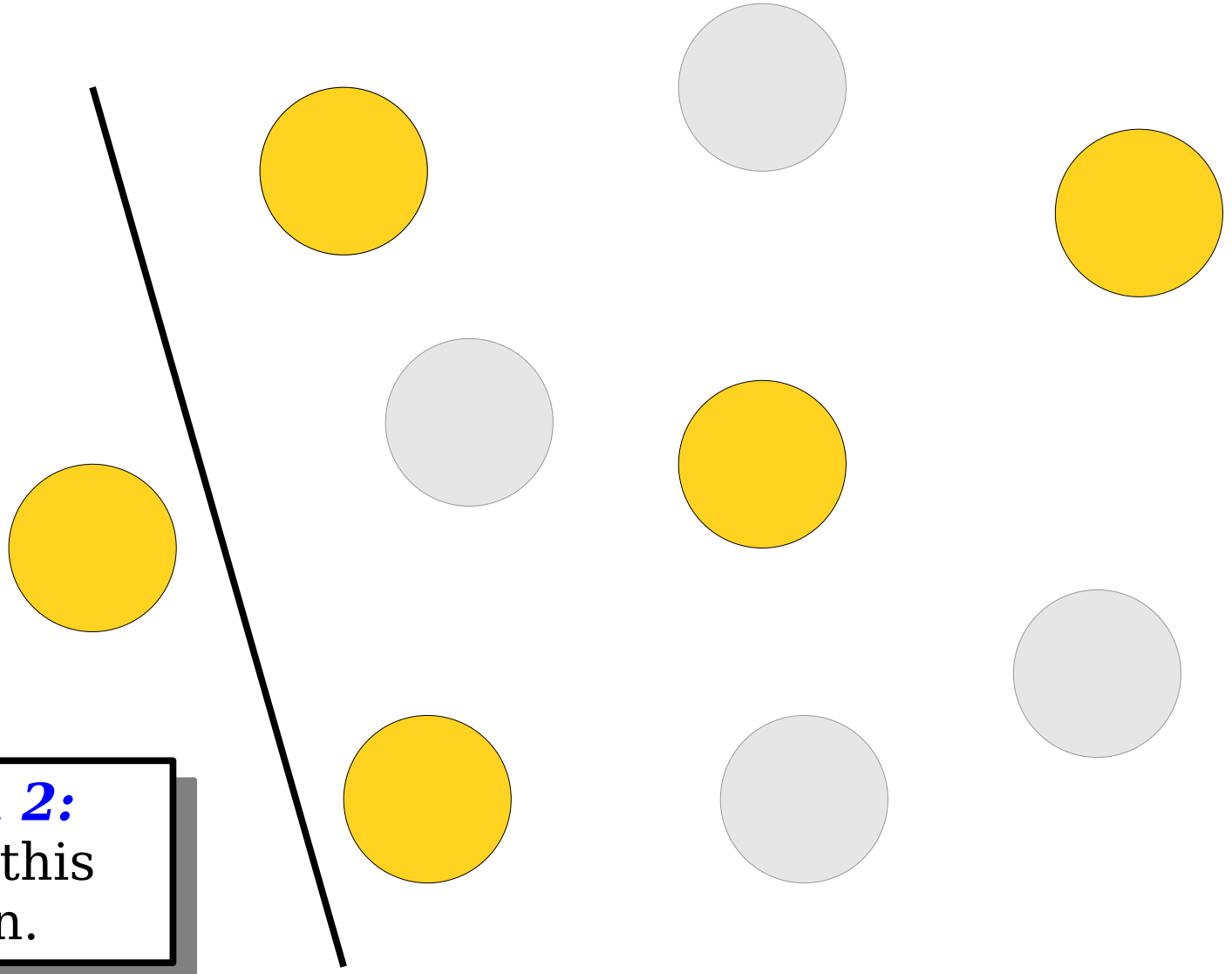

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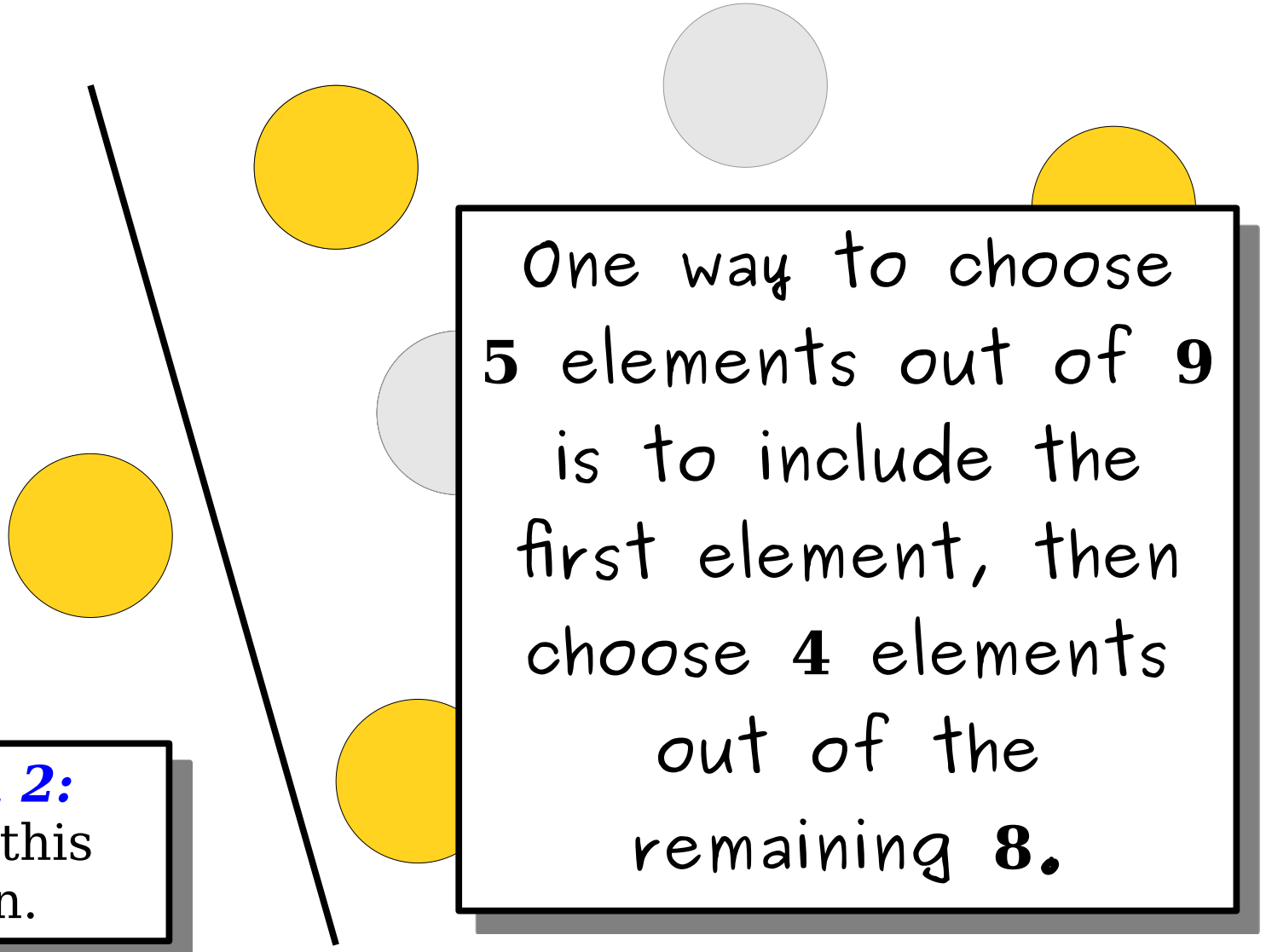


One way to choose **5** elements out of **9** is to include the first element, then choose **4** elements out of the remaining **8.**

Option 2:
Include this person.

Generating Combinations

- Base Case 1: If we need to pick zero more people, the only combination we can make is the one we've built up so far.
- Base Case 2: If we need to pick more items than what remains, we cannot make any combinations.
- Recursive Case: Pick an item. Then either include it (and we need one fewer item) or exclude it (and we still need the same number of items.)

A Comment on Types

The Wonderful `auto` Keyword

- There are many cases in which there is exactly one possible type that a variable could have.
- In that case, rather than explicitly writing out the type, you can use the `auto` keyword:

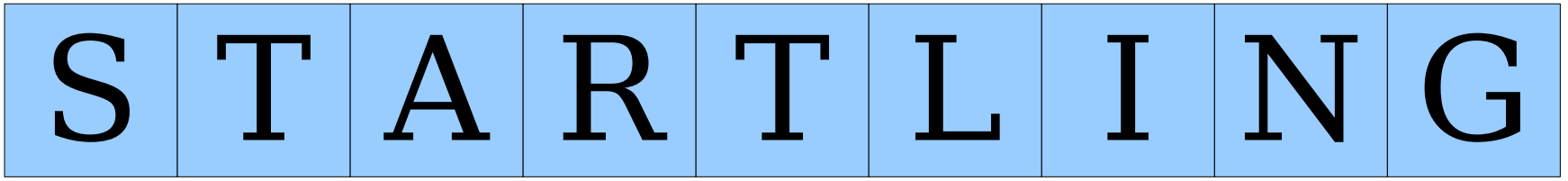
```
auto var = expression;
```

- While in principle you can use this in many places, we recommend just using it to save typing when working with container types.

A Little Word Puzzle

“What nine-letter word can be reduced to a single-letter word one letter at a time by removing letters, leaving it a legal word at each step?”

The Startling Truth?



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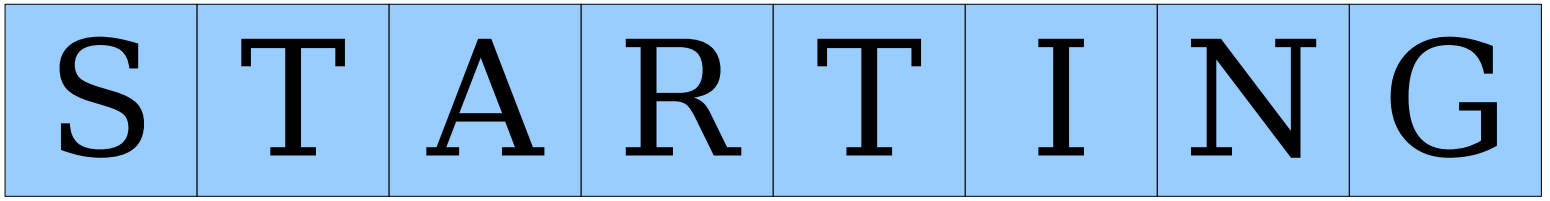

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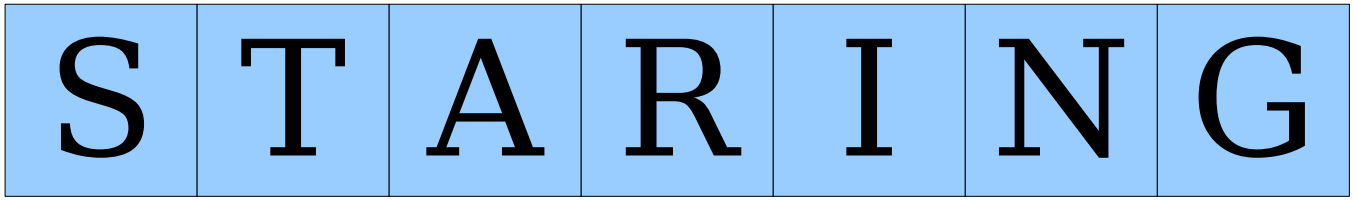

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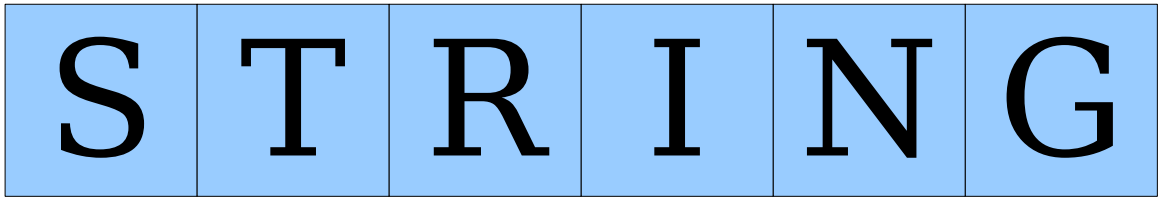

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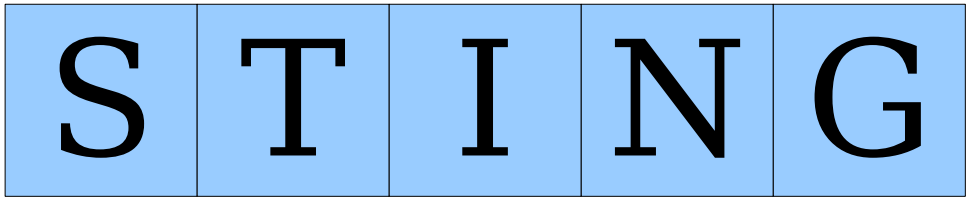

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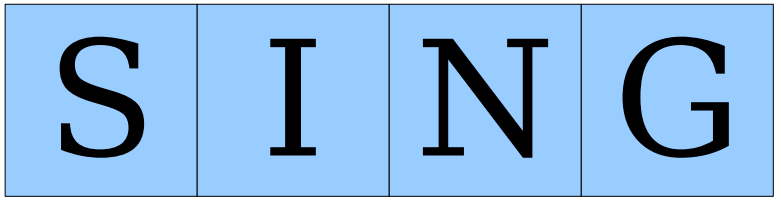

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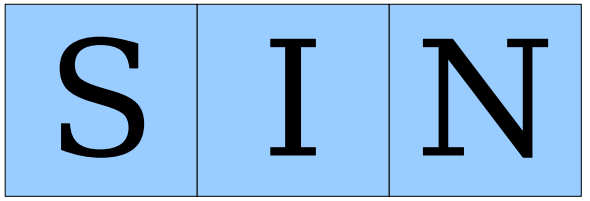

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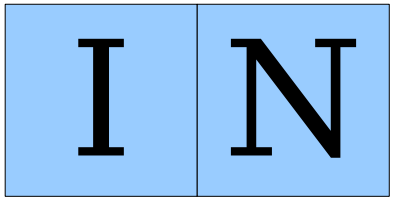

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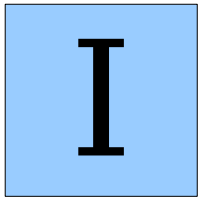
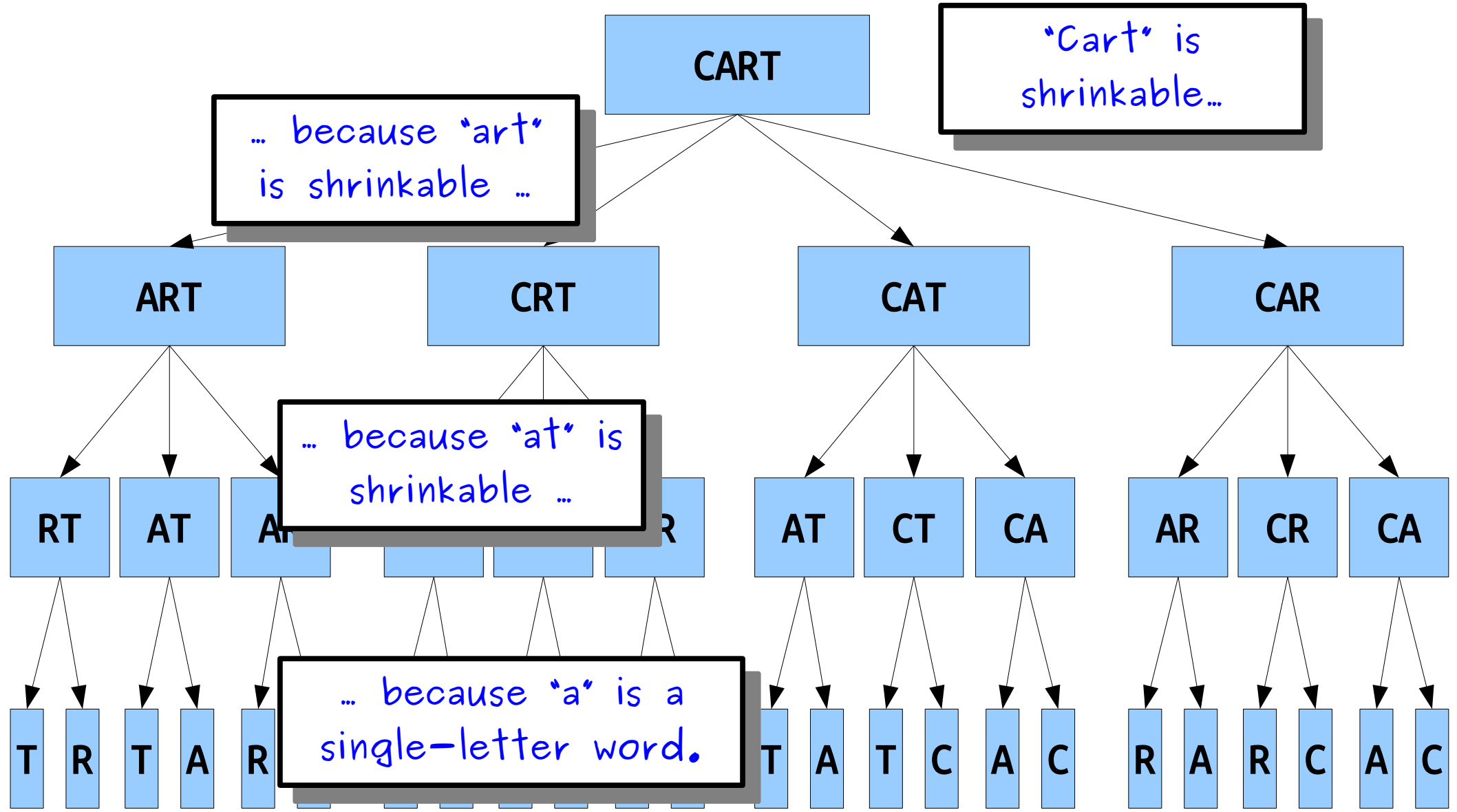
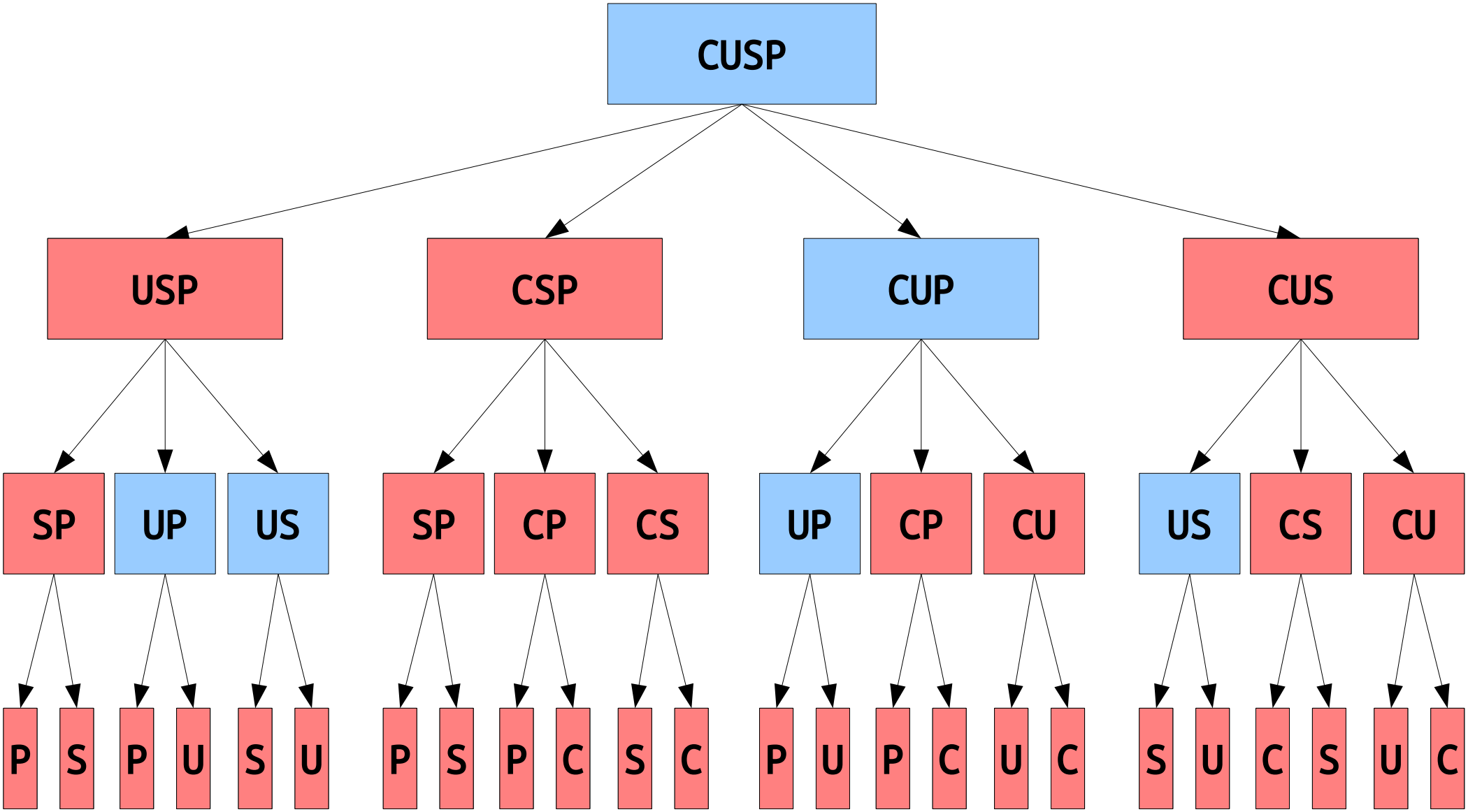


Is there ***really*** just one nine-letter word with this property?

All Possible Paths



All Possible Paths



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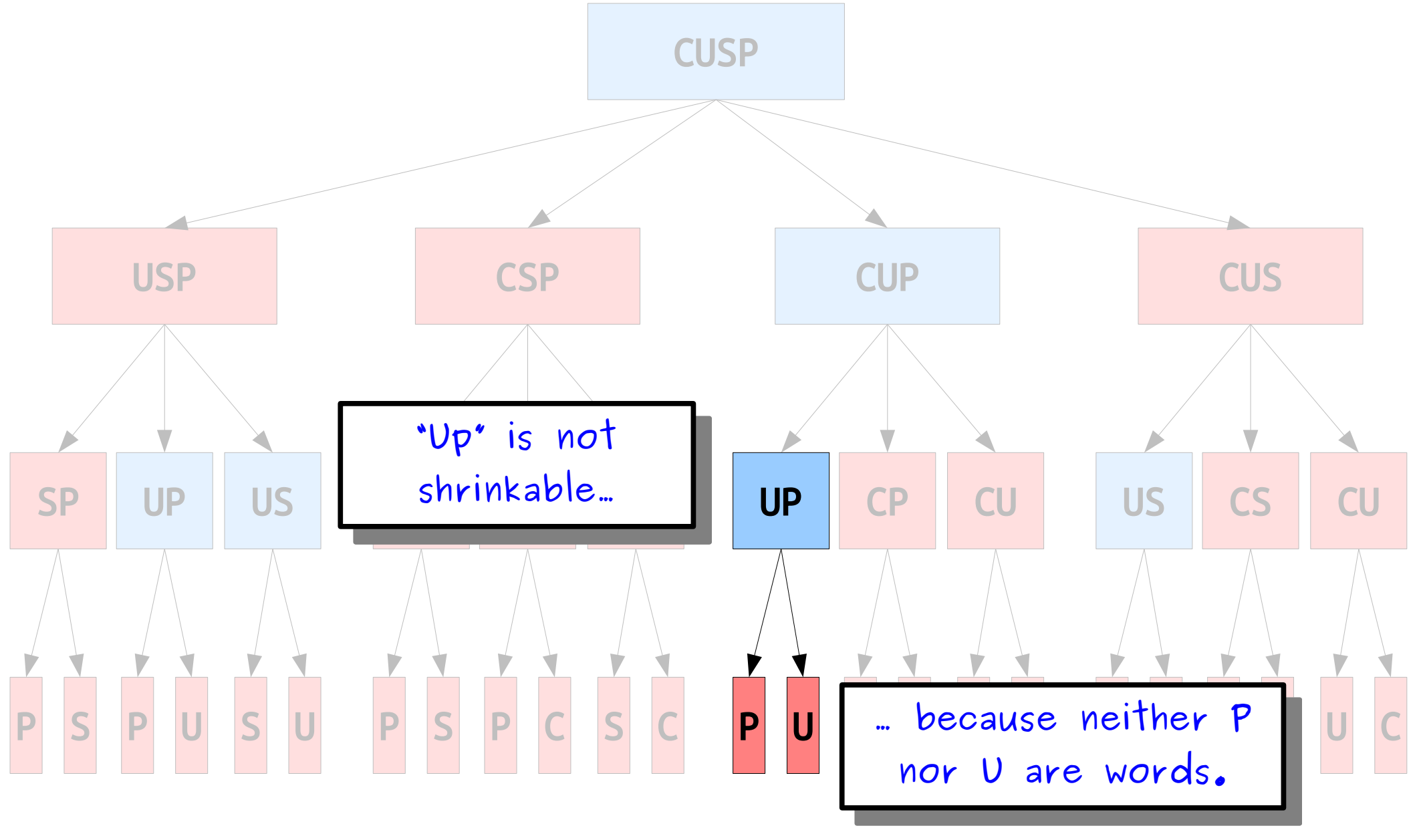

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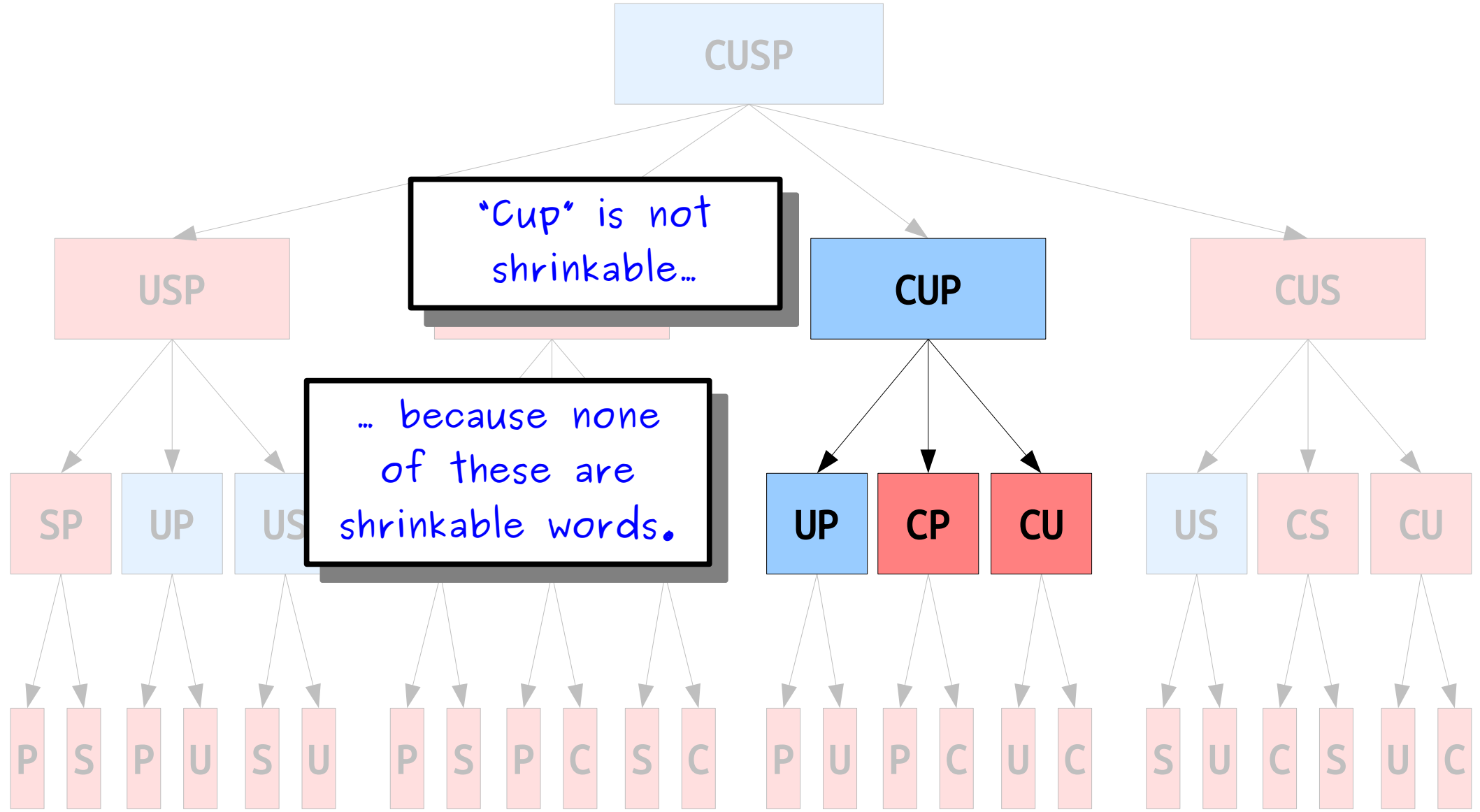

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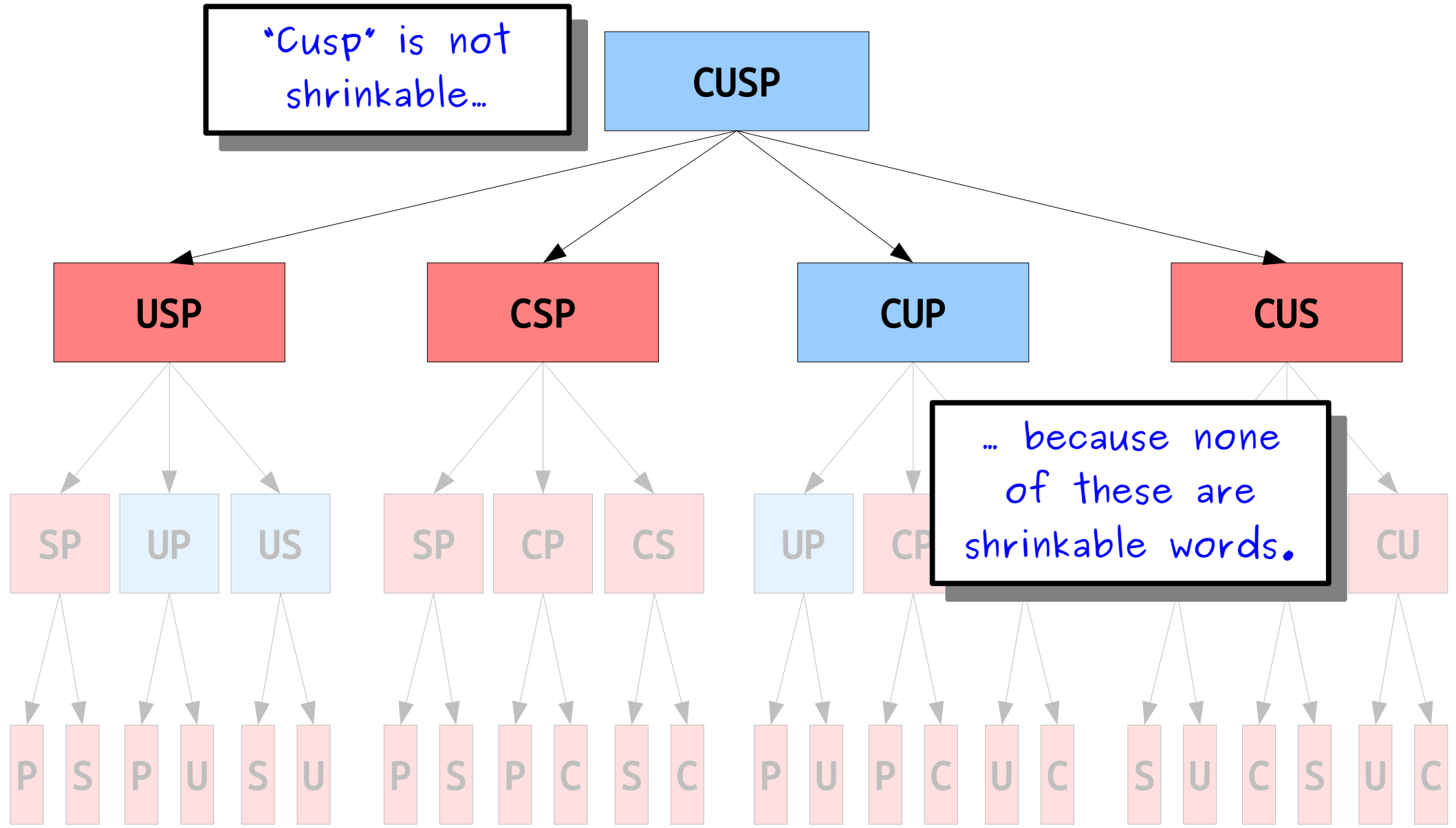

Shrinkable Words

- A ***shrinkable word*** is a word that can be reduced down to one letter by removing one character at a time, leaving a word at each step.
- ***Base Cases*:**
  - A string that is not a word is not a shrinkable word.
  - Any single-letter word is shrinkable (A, I, and O).
- ***Recursive Step*:**
  - A multi-letter word is shrinkable if you can remove a letter to form a shrinkable word.
  - A multi-letter word is not shrinkable if no matter what letter you remove, it’s not shrinkable.

Your Action Items

- ***Read Chapter 9 of the textbook.***
  - There's tons of cool backtracking examples there, and it will help you prep for Friday.
- ***Keep working on Assignment 3.***
  - If you're following our recommended timetable, you'll have finished Towers of Hanoi and Human Pyramids by the end of today and will have started Protein Synthesis.
  - Ask for help if you need it! That's what we're all here for.

Next Time

- ***Output Parameters***
  - Recovering the solution to a backtracking problem.
- ***More Backtracking***
  - Techniques in searching for feasibility.
- ***Closing Thoughts on Recursion***
  - It’ll come back, but we’re going to focus on other things for a while!